

國立中央大學 108 學年度碩士班考試入學試題

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所別：機械工程學系 碩士班 製造與材料組(一般生)  
機械工程學系 光機電工程 碩士班 光機組(一般生)  
能源工程研究所 碩士班 不分組(一般生)

科目：工程數學

本科考試可使用計算器，廠牌、功能不拘

計算題需計算過程，無計算過程者不予計分

1.

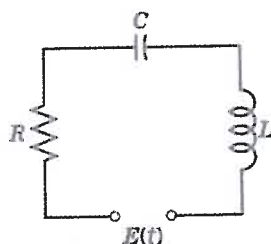
(a) Find the solution for the following ordinary differential equations (ODEs):

(i)  $yy'' = 3y'^2$ . (5%)

(ii)  $x^2y'' - xy' + y = 0, y(1) = 1.5, y'(1) = 0.25$  (5%)

(iii) Solve the following integration equation by the Laplace transform,  
 $y(t) + 4 \int_0^t y(\tau)(t - \tau)d\tau = 2t$  (5%)

(b) Referring to the following figure, find the steady-state current in the RLC-circuit when  $R = 50 \Omega, L = 30 \text{ H}, C = 0.025 \text{ F}$ , and  $E = 200 \sin 4t \text{ V}$ . (10%)



2.

A function  $f(t)$  is defined as:  $f(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 \leq t \leq 3 \\ 0, & 3 < t \end{cases}$

(a) Please write the Laplace transformation form of  $f(t)$  as  $F(s)$ . (10%)

(b) If  $g(t) = 2e^{-2t}$ , for  $t \geq 0$ , please find  $h(t)$  for  $t \geq 0$ , where

$h(t) = \int_{-\infty}^{+\infty} g(t - \tau)f(\tau)d\tau$  (15%)

參考用

注意:背面有試題

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3.

(a) At the beginning ( $t = 0$ ), we struck a string of length  $L$  by a flat hammer of width  $2d$  centred at the point  $\xi$ , having velocity  $v$ , with the wave speed  $c$ . Could you please describe the ensuing motion of the string? (5 %)

(b) Suppose the hammer is curved, rather than flat as above, so that the initial velocity distribution is demonstrated in the following,

$$u_t(x, 0) = \begin{cases} v \cos\left(\frac{\pi(x - \xi)}{2d}\right), & |x - \xi| < d \\ 0, & |x - \xi| > d \end{cases}$$

Please find the ensuing motion.

(10 %)

(c) Compare the kinetic energies of each harmonic in the two solutions. Where should the string be struck in order to maximise the energy in the  $n^{\text{th}}$  harmonic in each case? (10 %)

4.

(a) The temperature in a rectangular box can be approximated by  $T(x, y, z) = xyz(1 - x)(2 - y)(3 - z)$ ,  $0 \leq x \leq 1$ ,  $0 \leq y \leq 2$ ,  $0 \leq z \leq 3$ . If a mosquito is located at  $(1/2, 1, 1)$ , in which direction should it fly to cool off as rapidly as possible? **Hint:** evaluate the gradient of  $T(x, y, z)$  (10%)

(b) Please diagonalize the matrix,  $A = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{vmatrix}$  by evaluating both eigenvalues and eigenvectors

(8%); Please also evaluate  $A^{10}$  (7%)

注意:背面有試題

參考用