國立中央大學 110 學年度碩士班考試入學試題

所別： 通訊工程學系碩士班 不分組(一般生)

科目： 工程數學(線性代數、機率)

本科考試禁用計算器 ＊請在答案卷(卡)內作答

計算題 ※計算題需計算過程，無計算過程者不予計分

1. (10%) Let A and B be 3×3 matrices with $\det(A) = 5$ and $\det(B) = -6$. Find the value of:
   (a) (5%) $\det(2AB)$

   (b) (5%) $\det(A^{-1}B)$

2. (10%) For the following statements, please answer whether it is True or False. Explain your reasons.
   (a) (5%) If $A\mathbf{x} = B\mathbf{x}$ for some nonzero vector $\mathbf{x}$, then the matrices A and B must be equal.
   (b) (5%) If A is row equivalent to the identity matrix and $AB = AC$, then B must equal C.

3. (20%) Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 12 \\ 6 \\ 18 \end{bmatrix}$$

   (a) (5%) Use the Gram-Schmidt process to find an orthonormal basis for the column space of A.

   (b) (7%) Factor A into a product QR, where Q has an orthonormal set of column vectors and R is upper triangular.

   (c) (8%) Solve the least squares problem $A\mathbf{x} = \mathbf{b}$

4. (10%) Consider the matrix $A = \begin{bmatrix} x & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ with parameter x. Specify all numbers x, if any, for which A is positive definite. Explain your answer.

注意:背面有試題

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5.(8%) A box contains 10 identical balls numbered 1 through 10. Suppose 3 balls are drawn in succession.
(a)(4%) Find the probability that the smallest number drawn is more than or equal to 4.
(b)(4%) Find the probability that 8 is the largest number drawn.

6.(8%) We have three coins; the first two coins are fair and the last coin is two-headed. We pick one of the coins at random, and toss it twice. Heads show both times. Find the probability that the coin picked is fair.

7.(8%) The random variable $\mathbf{x}$ is uniform in the interval [-5, 5]. Define $\mathbf{y} = \begin{cases} \mathbf{x}, & \mathbf{x} \geq 0 \\ 0, & \mathbf{x} < 0 \end{cases}$. Find C.D.F. $F_{\mathbf{y}}(y)$ and p.d.f. $f_{\mathbf{y}}(y)$.

8.(10%) The joint p.d.f. of $\mathbf{x}$ and $\mathbf{y}$ is $f_{\mathbf{xy}}(x,y) = \begin{cases} k & 0 < x < y < 2 \\ 0 & \text{otherwise} \end{cases}$ for some k. Determine the conditional p.d.f. $f_{\mathbf{x|y}}(x|y)$ and $f_{\mathbf{y|x}}(y|x)$.

9.(16%) Let the joint p.d.f. of $\mathbf{x}$ and $\mathbf{y}$ be $f_{\mathbf{xy}}(x,y) = \begin{cases} 2e^{-(x+y)} & 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases}$. Define $\mathbf{z=x+y}$ and $\mathbf{w=y/x}$.
(a)(8%) Determine the joint p.d.f. of $\mathbf{z}$ and $\mathbf{w}$.
(b)(8%) Find the p.d.f. $f_{\mathbf{z}}(z)$ and $f_{\mathbf{w}}(w)$. Are $\mathbf{z}$ and $\mathbf{w}$ independent random variables?

注意:背面有試題