# 國立中央大學111學年度碩士班考試入學試題

所別： 數學系 碩士班 數學組(一般生)
數學系 碩士班 應用數學組(一般生)
數學系 碩士班 應用數學組(在職生)



科目： 線性代數

- Properly justify your answer to get full credits. Answers without sufficient reason may cause credits reduction.
- It is YOUR responsibility to provide clear and readable answers. Any unreadable answers will NOT be graded.

In this exam, $V$ and $W$ are assumed to be vector spaces over $\mathbb{R}$, the field of all real numbers. The set of all linear maps from $V$ to $W$ will be denoted by $\mathcal{L}(V, W)$. If $A$ is a matrix, then $A^t$ means the *transpose* of $A$.

1. (10pts) Find the determinant of the matrix

$$M = \begin{pmatrix} -3 & 0 & 1 & -5 & 7 \\ 1 & 7 & 6 & -1 & 1 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 4 & 2 & -3 & 0 \\ -4 & -8 & -4 & 6 & 2 \end{pmatrix}$$

2. Consider the following system of linear equations

$$\begin{cases} x_1 + 2x_2 - 3x_3 - 2x_4 + 4x_5 = 1 \\ 2x_1 + 5x_2 - 8x_3 - x_4 + 6x_5 = 4 \\ 3x_1 + 5x_2 - 7x_3 - 10x_4 + 12x_5 = -2 \end{cases}$$

(a.) (10pts) Find the reduced row echelon form of its augmented matrix $(A|b)$.

(b.) (10pts) Note that the vector $x = (21, -7, 0, 3, 0)^t$ is a particular solution for the system $Ax = b$. Use this given vector, explicitly write down the solution set $K$ of this linear system. Your answer should include a basis for its homogeneous solution set $K_H$.

3. (15pts) Consider the following matrix

$$A = \begin{pmatrix} 5 & 12 & 4 \\ -4 & -11 & -4 \\ 4 & 12 & 5 \end{pmatrix}$$

If $A$ is not diagonalizable, then give a convincing reason. Otherwise, find a diagonal matrix $D$ and an invertible matrix $Q$ such that $D = Q^{-1}AQ$.

注意:背面有試題

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**4.** Let $\mathbb{R}_2[x]$ be the set of polynomials of degree **less than or equal to** 2, which is a vector space over $\mathbb{R}$. Let $\beta = \{1, x, x^2\}$ be the standard ordered basis for $\mathbb{R}_2[x]$. Consider the map $T \in \mathcal{L}(\mathbb{R}_2[x], \mathbb{R}_2[x])$ given by

$$T(f(x)) = f(2) + f'(x)x + f''(1)x^2, \qquad \forall f(x) \in \mathbb{R}_2[x]$$

(a.) (5pts) Find the matrix $[T]_\beta$.

(b.) (5pts) Find the sum of all eigenvalues of $T$.

(c.) (5pts) Show that the linear map $T$ is invertible.

(d.) (10pts) Find $T^{-1}(3 + x + 4x^2)$.

**5.** (15pts) Let $V$ be an inner product space and let $W$ be a finite dimensional subspace of $V$. Let $x \notin W$ be given. Show that there exists some $y \in W^\perp$ such that $< x, y > \neq 0$. Here $W^\perp$ means the orthogonal complement of $W$ with respect to the inner product $<,>$.

**6.** (15pts) Find the singular value decomposition of $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

*Hint: You need to find orthogonal matrices $X$ and $Z$ such that $A = X \cdot Y \cdot Z^t$ for a certain matrix $Y$. They are related to eigenvalues of $AA^t$ and $A^tA$.*

注意:背面有試題