1. (26%) Consider the following closed-loop feedback system as depicted in Figure 1, where G_P and G_C denote the system plant and controller, respectively.

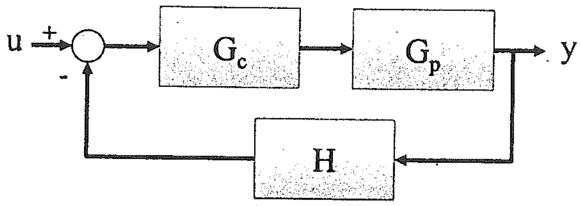

Figure 1

Let the Laplace transform of the error $e(t)$ be defined as $E(s)=U(s)-Y(s)\cdot H(s)$. Here, $U(s)$, $Y(s)$ and $H(s)$, respectively, denote the Laplace transform of the input, output and feedback block.

(a) (4%) Let $G_P=\dfrac{10}{s(s+3)}$, $G_C=K$ and $H(s)=1$. Find the value of K so that the closed-loop poles will have undamped natural frequency $\omega_n=10$. In addition, what will be the value of the corresponding damping ratio ζ?

(b) (6%) Plot the root loci for the closed-loop system defined in part (a) with $K\geq 0$ and find the range of K for guaranteeing the stability of the closed-loop system.

(c) (8%) Let $G_P=\dfrac{5}{s(s+2)}$ and $H(s)=1$. Design a controller $G_C(s)$ to make the closed-loop system to have the desired poles $p^*=-5\pm jb$. What will be the corresponding value of b?

(d) (8%) Let $G_P=\dfrac{5}{s(s+2)}$ and $H(s)=\dfrac{K}{(s+10)}$. Plot the root loci for the closed-loop system with the same $G_C(s)$ defined in part (c) for $K\geq 0$ and find the range of K for guaranteeing the stability of the closed-loop system.

注意:背面有試題

2. (24%) Consider the closed-loop feedback system as given in Figure 1 above with the characteristic equation of the closed-loop system being given by $s^3+(a+6)s^2+2Ks+2K=0$.

(a) (8%) Let $G_C=2K$ and $H(s)=1$. Then find the function $G_P(s)$ and plot the root loci of the closed-loop system with $a=10$ for $K\geq 0$.

(b) (4%) Consider the closed-loop system defined in part (a) with $K=5$ and $a=10$. Solve the steady-state error for input $U(s)=\frac{1}{s}$ and $U(s)=\frac{1}{s^2}$, respectively.

(c) (6%) Let the system plant $G_P(s)$ be the same as the one obtained in part (a) with the parameter a being a variable, $H(s)=1$ and $G_C=2K$. Find the value of a and the corresponding value of K so that the closed-loop system will have a triple poles on the real axis for $K\geq 0$.

(d) (6%) Let the system plant G_P be the same as the one obtained in part (a) with the parameter a being a variable, $H(s)=1$ and $G_C=2K$. Find the range of a so that the closed-loop system will have no non-zero breakaway and/or break-in points on the real axis for $K\geq 0$.

3. (24%) Consider the feedback system in Figure 2, where $C(s) = K > 0$ is a constant gain. Suppose that $G(s)$ has one unstable pole. For $K = 1$, the Nyquist plot of the system is shown in Figure 3. Note that the small circle in Figure 3, which is centered at $(-1,0)$ with radius $r_0 = 0.1544$, is tangent to the Nyquist plot. Answer the following questions.

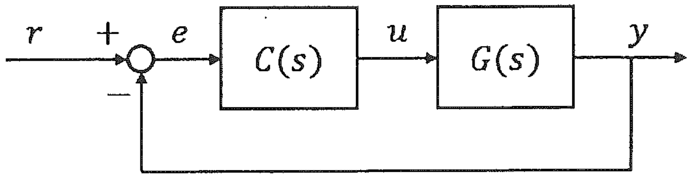

Figure 2: The feedback system for Problem 3 and Problem 4

(a) (4%) What are the gain margin and the phase margin of the system when $K = 1$?

(b) (6%) When K varies from 0 to ∞, find the number of unstable closed-loop poles for different ranges of K.

(c) (4%) For what value of K does the closed-loop system have a pair of pure imaginary poles?

(d) (4%) What is the steady-state error with respect to the unit-step input when $K = 0.8$?

(e) (6%) Let $r(t) = \sin(\omega t)$ and $K = 1$. The steady-state error is

$$e(t) = A(\omega)\sin(\omega t + \phi(\omega))$$

where the amplitude $A(\omega) > 0$ and the phase $\phi(\omega)$ are functions of ω. What is $\max_{\omega>0} A(\omega)$?

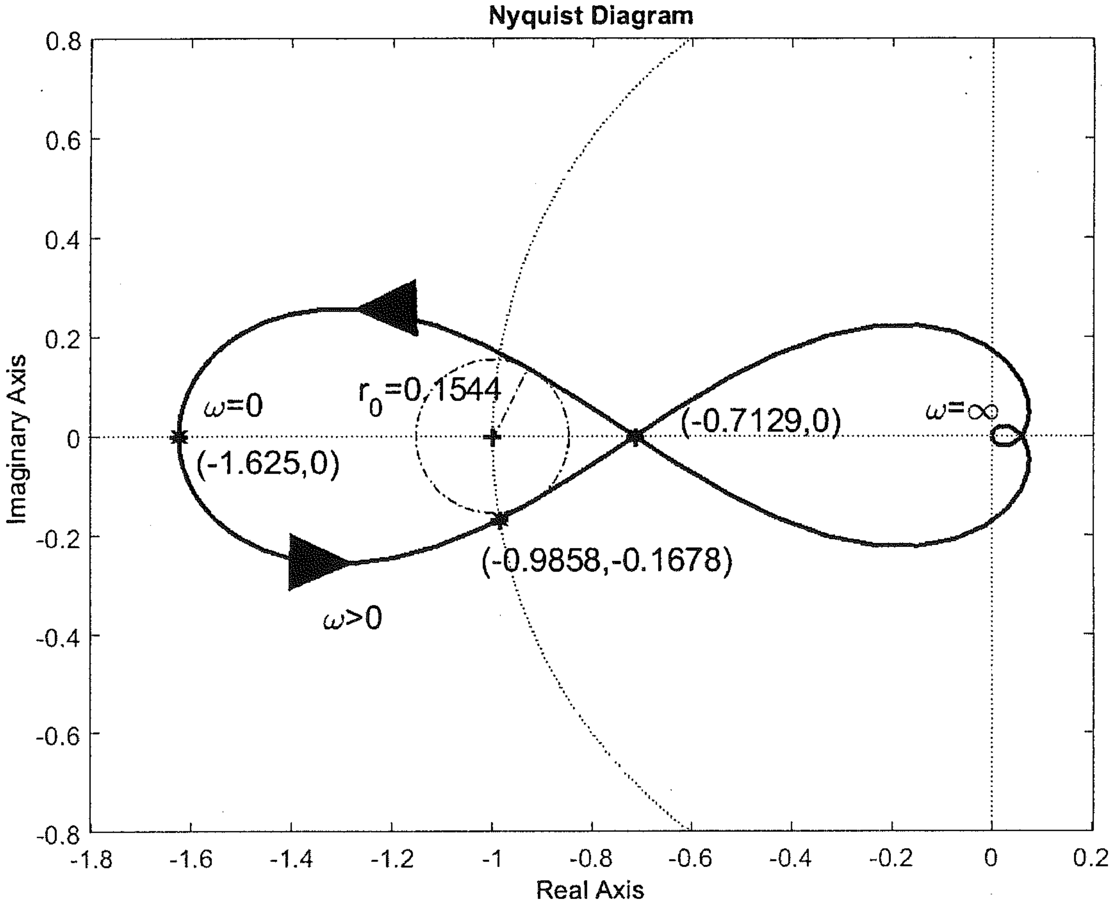

Figure 3: The Nyquist plot for Problem 3 when $K = 1$

4. (26%) Consider the feedback system in Figure 2, where

$$G(s) = \frac{2\sqrt{5}}{s(s+1)}, \qquad C(s) = K_p + \frac{K_I}{s}$$

$K_P, K_I > 0$. Define $z = \frac{K_I}{K_P}$.

(a) (6%) Find the conditions on K_p and K_I such that the closed-loop system is stable.

(b) (6%) Suppose that $K_p = 1$ and $K_I = 0$. Find the gain margin and phase margin of the system.

(c) (6%) Let K_p vary from 0 to ∞. Draw the root loci of the system for $z = 0.5$ and $z = 2$, respectively. Specify the intersection of asymptotes in each case.

(d) (8%) Sketch the Nyquist plots for $z = 0.5$ and $z = 2$. Suppose that the Nyquist path detours to the right around $s = 0$. Which closed-loop system ($z = 2$ or $z = 0.5$) is stable based on the Nyquist criterion?