

甲、填充題：共 8 題，每題 8 分，共 64 分。請將答案依題號順序寫在答案卷第一頁上。

請注意：本（甲、）部分，共 8 題，命題型態為填充題，請依題號順序獨立列出，勿同時陳列出計算過程。倘若答案被包含在演算過程，將被視為試算流程，不予另行挑出計分。

1. Evaluate $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n}\sqrt{n+n}} \right)$.

Answer : _____

2. Find the volume of the smaller region cut from the solid sphere $\rho \leq 2$ by the plane $z = 1$.

Answer : _____

3. Evaluate $\int \frac{4x^2 - 3x + 2}{4x^2 - 4x + 3} dx$.

Answer : _____

4. Evaluate the iterated integral $\int_0^1 \int_x^1 \sin(y^2) dy dx$.

Answer : _____

5. Find the area of the portion of the plane $y + 2z = 2$ inside the cylinder $x^2 + y^2 = 4$.

Answer : _____

6. Suppose that $F(x)$ is an antiderivative of $f(x) = \frac{\cos x}{x}$, $x > 0$. Express $\int_1^3 \frac{\cos 3x}{x} dx$ in terms of F .

Answer : _____

7. Along all triangles in the first quadrant formed by the x -axis, the y -axis, and tangent lines to the graph of $y = 3x - x^2$, what is the smallest possible area?

Answer : _____

8. A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters Earth's atmosphere and its surface begins to heat. After 1 hour, the temperature at the point (x, y, z) on the probe's surface is $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's surface.

Answer : _____

注意：背面有試題

乙、計算、證明題：共 3 大題，每大題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1 Find the Taylor polynomials of orders 2 generated by $f(x) = \begin{cases} e^{-1/x^2}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ at $a = 0$.

2. Let a and b be constants with $0 < a < b$. Does the sequence $\{(a^n + b^n)^{1/n}\}$ converge? If it does converge, what is the limit?

3. Find the limit of f or show that the limit does not exist.

a. (6 分) $\lim_{(x,y) \rightarrow (1,0)} \frac{xe^y - 1}{xe^y - 1 + y}$.

b. (6 分) $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x-y)}{|x| + |y|}$.