科目 微積分 類組別 A3/A4/A6 共2頁 第1頁

甲、填充題：共 8 題，每題 8 分，共 64 分。請在答案卷 上列出題號依序作答。

請注意：本（甲）部分，共 8 題，命題型態為填充題，請依題號順序獨立列出，勿同時陳列出計算過程。倘若答案被包含在演算過程，將被視為試算流程，不另行挑出計分。

1. Find the limit $\lim_{x\to 0+}\left(\cos\left(2\sqrt{x}\right)\right)^{\frac{2}{x}}$.

2. Find $f'(-1)$ if $f(x)=e^{g(x)}$ and $g(x)=\tan^{-1}\left(x^2\right)+\int_1^{x^2}\sec(t-1)\,dt$.

3. Find the length of the curve $x=\ln(\sec t+\tan t)-\sin t$, $y=\cos t$, $0\le t\le \pi/3$.

4. Consider the region bounded by the graphs of $y=\ln x$, $y=0$, and $x=e$. Find the volume of the solid formed by revolving the region about the x-axis.

5. Find the derivative of $f(x,y,z)=xyz$ in the direction of the velocity vector of the helix $\mathbf{r}(t)=(\cos 3t)\mathbf{i}+(\sin 3t)\mathbf{j}+3t\mathbf{k}$ at $t=\pi/3$.

6. Evaluate the integral $\int_0^8\int_{\sqrt[3]{x}}^2\frac{1}{y^4+1}\,dydx$.

7. Evaluate the integral $\iint_R\sqrt{x^2+y^2}\,dA$, where R is the region inside the upper semicircle of radius 2 centered at the origin, but outside the circle $x^2+(y-1)^2=1$.

8. Evaluate the integral $\int_{-1}^4\frac{dx}{\sqrt{|x|}}$.

注意：背面有試題

乙、計算、證明題：共 3 題，每題 12 分，共 36 分。須詳細寫出計算及證明過程，否則不予計分。

1. (a) (6 分) Determine whether the series $\sum_{n=2}^{\infty}(-1)^n\frac{\ln n}{n-\ln n}$ converges absolutely or converges conditionally or diverges and give reasons for your answer.

   (b) (6 分) Find all values of x for which $\sum_{n=1}^{\infty}\frac{(x+4)^n}{n3^n}$ converges and give reasons for your answer.

2. Let $f(x,y)=\begin{cases}\frac{x^2y}{x^3+y^3}, & x^3+y^3\neq 0\\ 0, & x^3+y^3=0\end{cases}$.

   (a) (4 分) Show that f is not continuous at $(0,0)$.

   (b) (4 分) Find the partial derivative $\frac{\partial f}{\partial x}$ at (x,y) if $x^3+y^3\neq 0$.

   (c) (4 分) Use the definition of the partial derivative to find $\frac{\partial f}{\partial x}$ at $(x,y)=(0,0)$.

3. Find the absolute maximum and minimum values of $f(x,y)=x^2+3y^2+2y$ on the unit disk $x^2+y^2\leq 1$.