科目：通訊系統(通訊原理)(300F)

校系所組：中央大學通訊工程學系（甲組）
中央大學電機工程學系（電子組）
交通大學電子研究所（乙B組）
交通大學電信工程研究所（甲組）
清華大學電機工程學系（乙組）
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一. (20%) For an analog modulation system with an output bandpass signal expressed by $x_c(t) = s_I(t)\cdot\cos(2\pi f_c t) - s_Q(t)\cdot\sin(2\pi f_c t)$ and a message signal $m(t)$ having the characteristics of $|m(t)| \le m_{\max}$ and $|M(f)| = |\Im\{m(t)\}| = \begin{cases} M_F, & 200\text{ Hz} \le |f| \le 4000\text{ Hz} \\ 0, & \text{otherwise} \end{cases}$, find $s_I(t)$ and $s_Q(t)$ (with minimum bandwidth $B_{\min}$) in terms of $m(t)$ and the minimum bandwidth $B_{\min}$ when

(一) (7%) $x_c(t)$ is an AM signal with 70% modulation;

(二) (6%) $x_c(t)$ is a Single Sideband signal;

(三) (7%) $x_c(t)$ is an FM signal with a maximum frequency deviation $f_D = 8\times 10^4$ Hz.

(Hint: The spectrum of $x_c(t)$ is located at $f_c - B_{\min} \sim f_c + B_{\min}$ when $s_I(t)$ and $s_Q(t)$ have the minimum bandwidth $B_{\min}$, $\Im\{\ \}$: Fourier transform, the FM bandwidth approximated with Carson's rule)

二. (20%) Consider a set of eight energy signals

$$s_0(t) = \begin{cases} \sqrt{2}\left(\cos(2\pi t) + \sin(2\pi t)\right) & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases},\quad s_1(t) = \begin{cases} \sqrt{2}\left(3\cos(2\pi t) + \sin(2\pi t)\right) & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases},$$

$$s_2(t) = \begin{cases} \sqrt{2}\left(\cos(2\pi t) - \sin(2\pi t)\right) & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases},\quad s_3(t) = \begin{cases} \sqrt{2}\left(3\cos(2\pi t) - \sin(2\pi t)\right) & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases},$$

and $s_i(t) = -s_{i-4}(t), i = 4,\cdots,7$. The eight signals are equally likely to be transmitted.

(一) (7%) Please draw the constellation points for the above signals $s_i(t)$ for $i = 0,\cdots,7$, if the basis functions are

$$\phi_1(t) = \begin{cases} \sqrt{2}\cos(2\pi t) & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases} \text{ and } \phi_2(t) = \begin{cases} \sqrt{2}\sin(2\pi t) & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

(二) (5%) Please calculate the average energy.

(三) (8%) Please draw block diagrams of the correlation receiver and the equivalent matched filter receiver for the optimum demodulation of the signals in an AWGN channel. Specify the impulse responses of the matched filters and the output sampling time.

注意：背面有試題

參考用

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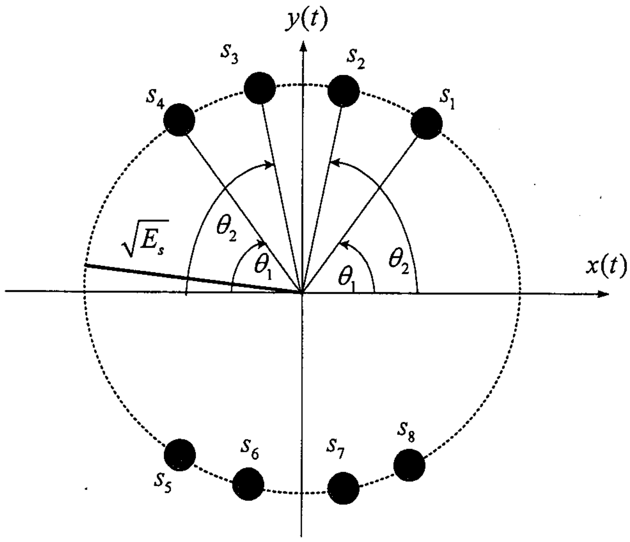
三. (20%) Consider the following 8-ary modulation scheme, and *x(t)* and *y(t)* are orthonormal functions. Note that all four quadrants are symmetrical and Gray coding is used.



(一) (5%) What is the spectral efficiency of this modulation scheme (bits/symbol)?

(二) (10%) Assume that the optimum coherent receiver is used. Determine the approximate bit error rate (BER) in an AWGN channel in terms of E_b/N_o (assume that all symbols are equally likely, and all answers should be presented as Q-function), given $\theta_1 = \pi/3$ and $\theta_2 = 4\pi/9$.

(三) (5%) If we wanted to minimize BER, what would be the optimal values of θ_1 and θ_2?

(Hint:

- Formally, the Q-function is defined as $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$.
- Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$. $\cos\frac{\pi}{9}$=0.94, $\cos\frac{2\pi}{9}$=0.77, $\cos\frac{3\pi}{9}$=0.50, $\cos\frac{4\pi}{9}$=0.17.)

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四. (20%)

(一) (6%) Consider a signal set which contains eight signals $s_i(t) = a_i \cos(2\pi f_c t + \frac{\pi}{2} \times \left\lfloor \frac{i-1}{2} \right\rfloor + \frac{\pi}{4}), 0 \le t \le T_s$ for i=1,2,…,8, where $a_i = \begin{cases} r_0 & \text{if } i \text{ is odd} \\ r_2 & \text{if } i \text{ is even} \end{cases}$ $(r_2 > r_0)$, $\lfloor x \rfloor$ denotes the integer part of x, and T_s denotes the symbol interval. Please determine the value of $r = \frac{r_2}{r_0}$ which maximizes the minimum Euclidean distance under the constraint $r_0^2 + r_2^2 = 2$.

(二) (6%) To increase bandwidth efficiency, additional eight signals $s_i(t) = r_1 \cos(2\pi f_c t + \phi_i), 0 \le t \le T_s$ for i=9,10,…, 16, are added into the signal set, where $r_0 < r_1 < r_2$ and $\phi_i = \begin{cases} \frac{\pi}{4}(i-9) + \tan^{-1}\frac{1}{3} & \text{if } i \text{ is odd} \\ \frac{\pi}{4}(i-8) - \tan^{-1}\frac{1}{3} & \text{if } i \text{ is even} \end{cases}$.

Please determine the value of r_1 which maximizes the minimum Euclidean distance under the constraint $\frac{r_2}{r_0} = 3$ and $r_0^2 + r_1^2 + r_2^2 = 3$.

(三) (8%) Consider another signal set which contains sixteen signals

$x_i(t) = p_i \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t) + q_i \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t), 0 \le t \le T_s$ for i=1,2,…,16. At the transmitter, four equally-likely data bits u_0, u_1, u_2, u_3 determine the values of p_i and q_i according to $p_i = -3 + u_1 \times 4 + u_0 \times 2$ and $q_i = -3 + u_3 \times 4 + u_2 \times 2$. The received signal is $x_i(t) + n(t)$ where $n(t)$ is the zero-mean white Gaussian noise with two-sided power spectral density $N_0/2$. Assume that the optimum coherent receiver is used. Please find the bit error probability expressed in terms of the Q function defined as $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$.

(Hint: Consider the in-phase channel and the quadrature channel independently.)

五. (20%) Consider a discrete memoryless source whose source output is modeled as a discrete random variable X, which takes on symbols from a fixed finite *alphabet* $\{a_0, a_1, \cdots, a_{J-1}\}$ with probabilities $P(X = a_j) = p_j, j = 0,1,\ldots, J-1$.

(一) (10%) Please show that the entropy $H(p)$ of such a source is bounded as follows:

$$0 \le H(p) \le \log_2 J \quad \text{where} \quad H(p) \equiv -\sum_{j=0}^{J-1} p_j \cdot \log_2 p_j .$$

(二) (10%) Please show that the entropy $H(p)$ is a concave function, i.e., for any two probability distribution functions p' and p'', the entropy satisfies

$$H(\lambda p' + \bar{\lambda} p'') \ge \lambda H(p') + \bar{\lambda} H(p'') \quad \text{with} \quad \bar{\lambda} = 1 - \lambda, \ 0 \le \lambda \le 1.$$

(Hint: Note that $\lambda p' + \bar{\lambda} p''$ is another probability distribution function.)