科目: 工程數學 C(3005)

参考

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- 請將答案依下圖所示由上而下依序寫在答案卷的作答區的第一頁。
- 只要填寫考題所要求的答案,請勿附加計算過程。

從此處開始寫起		
一、(一) · · ·	(二)· · ·	
二、(一)	(_)···	
三、(一) · · ·	(-)	
四、(一)	(-)	
五、(一)	(二)···	
:		

- $-\cdot$ (9 %) Let a > b > 0, and define $\mathbf{M} = a\mathbf{u}\mathbf{u}^{\mathsf{T}} + b\mathbf{I}_4$, where $\mathbf{u} = \begin{bmatrix} 1 & 2 & 2 & 1 \end{bmatrix}^{\mathsf{T}}$ and \mathbf{I}_4 is the identity matrix of dimension 4×4 .
 - (-) (5 %) Let $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ be the eigenvalues of **M**. Compute $\lambda_1 + \lambda_2 \lambda_3 \lambda_4$.
 - ($\stackrel{-}{-}$) (4%) Find the maximal singular value of matrix M.
- = \(\cdot(8\%)\) Assume that $(U, \langle \cdot, \cdot \rangle_U)$ and $(V, \langle \cdot, \cdot \rangle_V)$ are two Euclidean inner product spaces spanned by the ordered orthonormal bases $B_U = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $B_V = \{\mathbf{v}_1, \mathbf{v}_2\}$, respectively. Let $T: U \to V$ be a linear transformation from U into V, and the matrix representation of T with respect to B_U and B_V is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & -1 \end{bmatrix}.$$

- (-) (3 %) Let $\mathbf{u} = \mathbf{u}_1 \mathbf{u}_2$ and $\mathbf{v} = T(\mathbf{u})$. Compute the norm $\|\mathbf{v}\| = \sqrt{\langle \mathbf{v}, \mathbf{v} \rangle_V}$.
- (=) (5%) Find the $\mathbf{u}_0 \in U$ such that (i) $T(\mathbf{u}_0) = \mathbf{v}_1 + 3\mathbf{v}_2$ and (ii) $\|\mathbf{u}_0\| = \sqrt{\langle \mathbf{u}_0, \mathbf{u}_0 \rangle_U}$ is minimized.
- = \cdot (-) (4\%) Compute the projection matrix **P** and the projection vector **p** of **b** onto the column space of **A**, where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}.$$

注:背面有試題

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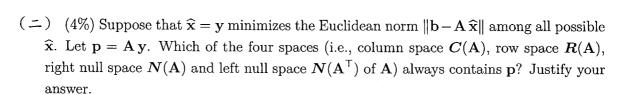
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四、 (-) (4%) Find the determinant of $(AB)^{-2}$:

$$\mathbf{A} = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}, \quad \mathbf{B} = \mathbf{L}\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 \\ \tan(\theta) & 1 & 0 \\ \sin(\theta) & \cos(\theta) & 1 \end{bmatrix} \begin{bmatrix} \theta & \theta^2 & \theta^3 \\ 0 & 2 & 2\theta \\ 0 & 0 & 4\theta^{-1} \end{bmatrix}.$$

(=) (4%) Give the property of block determinant:

$$\det \left(\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{D} \end{bmatrix} \right) = \det(\mathbf{A}) \cdot \det(\mathbf{D})$$

where A, B and D are some square matrices.

Use the above property to compute the determinant of the matrix:

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & 4 \\ 9 & -3 & 25 & 15 \\ 1 & -5 & 12 & -16 \end{bmatrix}.$$

Hint: Change it to the form
$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & m & n \\ 0 & 0 & p & q \end{bmatrix}$$
.

- $\underline{\mathcal{H}}$. (-) (4%) Given a 4×4 matrix $\mathbf{P} = \mathbf{P}_{21}\mathbf{P}_{31}\mathbf{P}_{41}\mathbf{P}_{32}$, where \mathbf{P}_{ij} is the basic 4×4 permutation matrix which exchanges row i with row j, find \mathbf{P}^{-1} .
 - ($\stackrel{\frown}{=}$) (4%) If a 3 × 4 matrix **A** has the vector [4, 8, 1, 0]^{\top} as the only special solution to $\mathbf{A}\mathbf{x} = \mathbf{0}$, find the reduced row echelon matrix **R** of **A**.
- $\dot{\pi}$, (-) (4%) Consider an arbitrary $m \times n$ matrix **A** with rank r, r < m and r < n, and an arbitrary column vector **b** of m elements. Find the number of all possible solutions to the general linear equation $\mathbf{A}\mathbf{x} = \mathbf{b}$.

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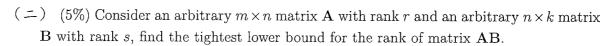
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 \pm \((8\%) Let $\delta(t)$ be the delta function. Solve the following equations and plot x(t).

$$(-)$$
 (4%) $x''(t) + 4x(t) = 4 \sum_{n=0}^{\infty} \delta(t - n\pi), x(0) = x'(0) = 0.$

$$(=)$$
 (4%) $\int_0^t x(t-u)\cos(u) du = t\sin(t)$.

へ、 (10%) Consider the following linear system

$$\mathbf{x}'(t) = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}.$$

(–) (3%) Find the complementary solution of the associated homogeneous system, i.e., if $f_1(t) = f_2(t) = 0$

(=) (4%) If $f_1(t) = f_2(t) = t^{-1}$, find the particular solution.

(
$$\equiv$$
) (3%) Given the initial condition $\mathbf{x}(1) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ in \wedge , (\equiv), find the solution $\mathbf{x}(t)$.

九、 (6%) Consider the following initial value problem

$$y''(x) - 2xy'(x) + 2y'(x) + 8y(x) = 0, \quad y(1) = 3, \ y'(1) = 0$$

Assuming $y(x) = \sum_{n=0}^{\infty} c_n x^n$ has a power series solution at x = 0, determine the values of $c_0, c_1, c_2, c_3, c_4, c_5$.

+ · (10%) Consider the following boundary value problem for u(x,t) with $0 < x < \pi$ and t > 0

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad u(0,t) = u(\pi,t) = 0, \quad u(x,0) = f(x), \quad \frac{\partial u}{\partial t}\Big|_{t=0} = 0. \tag{1}$$

(-) (5%) Assuming f(x) = x, determine its Fourier sine series on interval $[0, \pi)$.

(=) (5%) Solve the boundary value problem (1) for u(x,t) when f(x)=x.

+- \(\((16\)\)) Consider the following initial value problem

$$y''(x) + ay'(x) + by(x) = f(x), \quad y(0) = c, \ y'(0) = d,$$

where a, b, c, and d are constants. It is known that $y(x) = \sin(2x)$ when $f(x) = -3\sin(2x)$.

(-) (4%) Find the values of a, b, c, and d.

(=) (12%) Calculate the corresponding solution y(x) for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ when $f(x) = \tan(x)$.