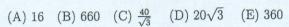
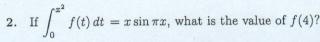
*請在試卷、答案卡內作答

甲、選擇題:共 10 題,每題 4 分,共 40 分。請用大寫字母 A,B,C,D 或 E 答題,並將答案依題號順序寫在答案卷上。皆單選。

1. What is the maximum value |f''(x)| for the function $f(x) = x^3(10 - 3x^2)$ on the closed interval [0, 2]?





(A) 0 (B)
$$-2\pi$$
 (C) 2π (D) $-\frac{\pi}{2}$ (E) $\frac{\pi}{2}$

3. What is the value of $\lim_{n\to\infty} \int_0^1 \frac{ny^{n-1}}{1+y} dy$?

(A)
$$\frac{1}{2}$$
 (B) $-\frac{1}{2}$ (C) 0 (D) $+\infty$ (E) $-\infty$

4. Which of the following series diverges?

(A)
$$\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$$
 (B) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$ (C) $\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$ (D) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2 + n} - n)$ (E) $\sum_{n=1}^{\infty} \sin \frac{1}{n^2}$

5. What is the value of the definite integral $\int_0^1 \frac{x^3 e^{x^2}}{(x^2+1)^2} dx$?

(A)
$$\frac{2e-5}{8}$$
 (B) $\frac{e-2}{4}$ (C) $\frac{e-1}{2}$ (D) $e-1$ (E) None of the above

6. Which function has maclaurin series $\sum_{n=0}^{\infty} (-1)^n 2^n x^n$?

(A)
$$\tan^{-1} 2x$$
 (B) $\ln(1+2x)$ (C) $\ln\left(\frac{1+x}{1-x}\right)$ (D) $\frac{1}{1+2x}$ (E) $\sin^{-1} 2x$

7. What is the area that lies inside the circle r = 6 and above the line $r = 3 \csc \theta$?

(A)
$$12\pi - 9\sqrt{3}$$
 (B) $36\pi - 9\sqrt{3}$ (C) $12\pi + \sqrt{3}$ (D) $12\pi - \sqrt{3}$ (E) $36\pi - \sqrt{3}$

8. What is the value of the double integral $\int_0^{\ln 10} \int_{e^x}^{10} \frac{1}{\ln y} \, dy dx$?

9. Which of the following curve is tangent to the surface $x^2 + y^2 - z = 1$ when t = 1?

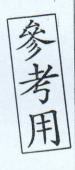
(A)
$$\mathbf{r}(t) = (2t - 1)\mathbf{i} + \sqrt{t}\,\mathbf{j} + \sqrt{t}\,\mathbf{k}$$
 (B) $\mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + (2t - 1)\mathbf{j} + \sqrt{t}\,\mathbf{k}$

(C)
$$\mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + \sqrt{t}\,\mathbf{j} + (2t - 1)\,\mathbf{k}$$
 (D) $\mathbf{r}(t) = t\,\mathbf{i} + t\,\mathbf{j} + \sqrt{t}\,\mathbf{k}$ (E) None of the above

10. Let S be the surface parametrized by $\mathbf{r}(\theta, z) = 3\sin 2\theta \,\mathbf{i} + 6\sin^2\theta \,\mathbf{j} + z \,\mathbf{k}$ where $0 \le \theta \le \pi$. Which of the following is equivalent to the surface differential $d\sigma$?

(A)
$$\frac{1}{3} d\theta dz$$
 (B) $d\theta dz$ (C) $3 d\theta dz$ (D) $6 d\theta dz$ (E) $12 d\theta dz$

注:背面有試題



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乙、填充題: 共 5 題, 每題 6 分, 共 30 分。請將答案依題號順序寫在答案卷上,不 必寫演算過程。

- 1. Find the limit: $\lim_{x\to 0^+} \left(\frac{\sin x}{x}\right)^{1/x^2}$. Answer:
- 2. Among all the points on the graph of parabolid $z = 10 x^2 y^2$ that above the plane x + 2y + 3z = 0, find the point farthest from the plane. Answer:
- 3. Set up the integral with the order $dzdrd\theta$ for evaluating the triple integral of the function $F(x,y,z) = \sin(z^2)$ over the solid region D bounded below by the plane z=0, laterally by the circular cylinder $(x-1)^2+y^2=1$, and above by the paraboloid $z=2-x^2-y^2$. Answer: (Do not evaluate the integral.)
- 4. Use the transformation (chang of variables) $x = uv^{-1}$, y = uv to evaluate

$$\iint_D (x^2 + y^2) \, dx dy$$

where $D = \{(x, y) : 1 \le xy \le 4, 1 \le y/x \le 4\}$. Answer:

5. At the point (1,2), the function f(x,y) has a derivative of 2 in the direction toward (2,2)and a derivative of -2 in the direction toward (1,1). Find the derivative of f at (1,2) in the direction toward the point (4,6). Answer:

丙、計算、證明題:共3題,每題10分,共30分。須詳細寫出演算過程,否則不 予計分。

- 1. Define $f(x) = \begin{cases} x^3 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$ Show that f'(x) is continuous at x = 0.
- 2. Find the values $a \leq b$ such that $\int_a^b 1 e^{(1-x^2)} dx$ has minimal value.
- 3. Let C be a simple colsed smooth curve in the plane x + 2y + 2z = 2. (Orient C to be counterclockwise when view from above.) Show that

$$\oint_C 3y\,dx - 2z\,dy + x\,dz$$

depends only on the area of the region enclosed by C and not on the position or shape of C.