所別：機械工程學系碩士班 甲組(固力與設計)(一般生)　科目：工程數學　共 2 頁 第 1 頁
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本科考試可使用計算器，廠牌、功能不拘　　＊請在試卷答案卷（卡）內作答

參考用

Vector analysis and linear algebra

1. An elastic membrane stretched in a specific direction can be described as an eigenvalue problem. Please consider a membrane with a boundary circle $x_1^2 + y_1^2 = 1$ is stretched from a point P:(x_1,y_1) to Q:(x_2,y_2) and experimentally determined by

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

(a) Please find the principal directions by solving eigenvalues and eigenvectors. **(10%)**

(b) Please show that the deformed boundary is an ellipse by using $\frac{x_2^2}{\lambda_1^2} + \frac{y_2^2}{\lambda_2^2} = 1$, where

$\lambda_1 \lambda_2$ are eigenvalues **(5%)**

2. Please use Gauss elimination method to solve the following linear systems of electrical networks

(a) $\begin{bmatrix} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 10 & 25 & 0 \\ 20 & 10 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 90 \\ 80 \end{bmatrix}$ **(5%)**

(b) $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$ **(5%)**

Ordinary differential equations

3. Find a general solution to the following ordinary differential equations (ODEs):

(a) $y' + xy = xy^{-1}$, $y(0) = -\frac{1}{3}$ **(5%)**

(b) $y'' - 2y' + y = 70x^{3/2}e^x$ **(5%)**

4. For a homogenous ODE given as $y''' + 2y'' - y' - 2y = 0$. (1)

(a) Find three solutions $y_1(x)$, $y_2(x)$, and $y_3(x)$ that can form a basis of solutions, show that they are linear independent, for Eq. (1). **(5%)**

(b) If there is a non-homogenous term $r(x)=1-4x^3$ of Eq. (1), then Eq. (1) becomes

$y''' + 2y'' - y' - 2y = 1 - 4x^3$, (2)

find the particular solution for Eq. (2), that is $y_p(x)$ = ? **(5%)**

(c) Transfer Eq. (2) to a system of 1st-order ODEs and express the 1st-order system of ODEs in a matrix form. **(5%)**

注意：背面有試題

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Partial differential equations and complex analysis

5. Solve

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\quad 0<x<1,\quad t>0,$$

$$u(0,t)=u_0(t),\quad u(1,t)=u_1(t),\quad t>0,$$

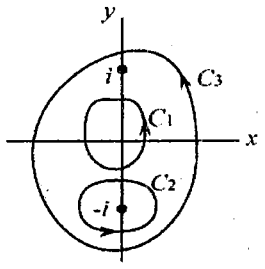
$$u(x,0)=h(x),\quad 0<x<1.$$

(a) For $u_0(t)=u_1(t)=0$, $h(x)=\sin \pi x$. **(5%)**

(b) For $u_0(t)=u_1(t)=0$, $h(x)=x$. **(5%)**

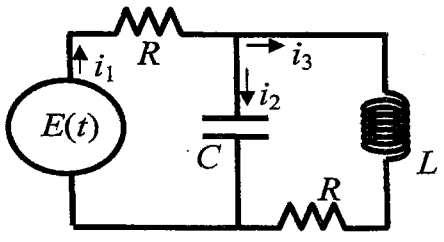
(c) For $u_0(t)=0$, $u_1(t)=\exp(-t)$, $h(x)=x$. **(5%)**

6. Evaluate the complex integral $\oint_C \frac{\sin z}{z^2+1}dz$ along the indicated closed contour, C_1, C_2 and C_3, respectively. **(10%)**



Laplace and Fourier transforms

7. The system of differential equations for the charge on the capacitor $q(t)$ and the current $i_3(t)$ in the electric network shown in Fig. 1 is



$$R_1\frac{dq}{dt}+\frac{1}{C}q+R_1 i_3=E(t)$$

$$L\frac{di_3}{dt}+R_2 i_3-\frac{1}{C}q=0$$

Figure 1

(1) **(10%)** If the applied voltage $E(t)$ can be formulated as

$$E(t)=\begin{cases}0, & t<0\\ e^{-t}, & t>0\end{cases}.$$

Express $E(t)$ in terms of Fourier Integral representations.

(2) **(15%)** Solve the charge on the capacitor when R_1=1Ω, R_2=1Ω, L=1h, C=1f,

$$E(t)=\begin{cases}0 & 0\le t<1\\ 50e^{-t} & t\ge 1\end{cases},$$

$i_3(0)$=0 and $q(0)$=0 by using Laplace transform.

注意：背面有試題