國立中央大學　資訊工程學系
九十二學年度 碩士在職進修專班 招生入學考試命題紙

科目：　離 散 數 學　　第一頁　共　頁

1. (25 Points)
A traveler is passing through a country where all the people are either Truth-Tellers, who always tell the truth, or Liars, who always lie. The traveler comes to a fork in the road, and at the fork of the road there is a man who is either a Truth-Teller or a Liar, but the traveler does not know which.
The traveler wants to go to a certain city. This time there are two forks in the road, a left fork, a right fork, and only one of these roads goes to the city.
The traveler must ask the man questions to determine which fork in the road leads to the city. What is the smallest number of questions that are needed? What should the questions be?

2. (25 Points)
Given f = { (a, b), (b, a), (c, b) }, a function from X = { a, b, c} to X:

(a) Write $f \circ f$ and $f \circ f \circ f$ as sets of ordered pairs.

(b) Define $f^n = f \circ f \circ f \circ .. \circ f$ to be the n-fold composition of f with itself. Find f^9 and f^{623}
*(*Definition of Composition of Functions: If $f: A \to B$ and $g: B \to C$ are functions, the composition of g and f is the function $g \circ f: A \to C$ defined by $(g \circ f) = g(f(a))$ for all $a \in A$.)*

3. (25 Points)

Let A = {1, 2, 4, 6, 8} and, for a, b ϵ A, define a ≤ b if and only if $\frac{b}{a}$ is an integer.

(a) Prove that ≤ defines a partial order on A

(b) Draw the Hasse diagram for ≤

(c) List all minimum, minimal, maximum and maximal elements.

(d) Is (A, ≤) totally ordered? Explain.

4. (25 Points)

(a) What is the maximum degree of a vertex in a graph with n vertices?
(b) What is the maximum number of edges in a graph with n vertices?
(c) How many edges must a Hamiltonian cycle in K_n contain?
(d) How many Hamiltonian cycles does K_n have? (Begin all cycles at the same vertex.)