國立中央大學　資訊工程學系
九十三學年度 碩士在職進修專班 招生入學考試命題紙

科目：　離 散 數 學　　第一頁 共　頁

1. (25 Points)

The Towers of Hanoi is a popular puzzle. It consists of three pegs and a number of discs of differing diameters, each with a hole in the center. The discs initially sit on one of the pegs in order of decreasing diameter (smallest at top, largest at bottom, thus forming a triangular tower. The object is to move the tower to one of the other pegs by transferring the discs to any peg one at a time in such a way that no disc is ever placed upon a smaller one.

(a) Solve the puzzle when there are n = 2 discs and show your moves by completing a little table like that below. [The pegs are labeled A, B, C, and we have used an asterisk (*) to denote an empty peg. The disks are numbered in order of increasing size, thus disk is the smallest.]

	A	B	C
Initial position	1, 2	*	*
Move 1			
Move 2			

(b) Give a recurrence relation for a_n, the number of moves required to transfer n discs from one peg to another.

(c) Find an explicit formula for a_n

2. (25 Points)

If ~ denote an equivalence relation on a set A, the equivalence class of an element $a \in A$ is the set $\bar{a}=\{ x \in A \mid x \sim a \}$.

Let A = { 1, 2, 3, 4, 5, 6, 7, 8, 9}. For a, b $\in$ A, define a ~ b if and only if a*b is a perfect square (that is, the square of an integer).

(a) What are the ordered pairs in this relation?

(b) For each $a \in A$, find $\bar{a}=\{ x \in A \mid x \sim a \}$

3. (25 Points)

(A) Define f: Z $\rightarrow$ Z by $f(x) = 3\ x^3 - x$. Is f onto? Is f one-to-one?

(B) (A) Define f: R $\rightarrow$ R by $f(x) = 3\ x^3 - x$. Is f onto? Is f one-to-one?

4. (25 Points)

Carling has three weeks to prepare for a tennis tournament. She decides to play at least one set every day but not more than 36 sets in all. Show that there is a period of consecutive days during which she will play exactly 21 sets.