國立中央大學100學年度碩士班考試入學試題卷

所別：機械工程學系碩士班 甲組(固力與設計)(一般生)　科目：工程數學　共 2 頁 第 1 頁
機械工程學系碩士班 乙組(製造與材料)(一般生)
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本科考試可使用計算器，廠牌、功能不拘　　＊請在試卷答案卷（卡）內作答

Ordinary Differential Equations

1. Find the general solution of the initial value problem

$$y''+6y'+25y=0;\ \ y(0)=2,\ \ y'(0)=3$$

Express the solution as a single function. (10%)

2. Show that $y_1=x$ and $y_2=x^2$ are both linearly independent solutions of $x^2y''-2xy'+2y=0$. (5%)

3. Consider the boundary value problem

$$\frac{d}{dx}\left[p(x)\frac{dy_k}{dx}\right]+\lambda_k q(x)y_k=0;\quad y_k(a)=y_k(b)=0,\ \ k=1,2,...n$$

where $p(x)$ and $q(x)$ satisfy the conditions stated above. Let the numbers λ_i and λ_j be the distinct eigenvalues of the given problem and corresponding solutions y_i and y_j be eigenfunctions. Show that $\int_a^b q(x)y_iy_jdx=0$. (10%)

Laplace/Fourier Transformation

4. Find the Fourier transform of $f(t)$, $f(t)=\exp(-a|t|)$, $a>0$. (5%)

5. (a) Find the Laplace transform of $g(t)$, $g(t)=\sin t+\delta(t-1)$, where $\delta(t-1)$ is the unit impulse function. (5%)
   (b) Using the Laplace transform, solve the initial value problem:

$$y''+3y'+2y=10g(t),\ \ y(0)=1,\ \ y'(0)=-1.$$ (15%)

Linear Algebra and Vector Calculus

6. Write down the expression for calculating the length of the closed curve $r=a+b\sin\theta$ in the $r\theta$ plane, where $a>b>0$ and $0\le\theta\le2\pi$. (5%)

7. Consider the linear system of equations $\mathbf{Ax}=\mathbf{b}$, where

$$\mathbf{A}=\begin{bmatrix}1&1&2\\2&1&3\\1&2&3\end{bmatrix},\quad \mathbf{b}=\begin{bmatrix}1\\m\\n\end{bmatrix},\ \text{and } m, n \text{ are real numbers.}$$

   (a) For what values of m and n will the system have solutions? (5%)
   (b) Solve $\mathbf{x}$ for the case of $m=0$, $n=3$. (5%)

參考用

注意：背面有試題

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8. Consider the matrix $\mathbf{B}=\begin{bmatrix} -1 & \sqrt{3} & 0 \\ \sqrt{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find the eigenvalues. (5%)
(b) Compute $\mathbf{B}^{10}$. (5%)

Partial Differential Equations and Complex Analysis

9. The two-dimensional wave equation expressing a circular membrane is modeled in polar coordinate system as

$$\frac{\partial^2 u}{\partial t^2}=\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right).$$

The boundary condition of the partial differential equation is $u(r=1, t)=0$.

(a) Assume the above partial differential equation can be solved by method of separating variables. Let $u(r,t)=R(r)T(t)$, find the two ordinary differential equations in which $R(r)$ and $T(t)$ are unknown functions, respectively, and also find the general solutions of the two ordinary differential equations. (10%)

(b) For the second mode (m=2) of the membrane, numerically determine the eigenvalue and the corresponding locations of nodal points. Namely, calculate the frequency of second mode and the corresponding radius r_o such that $u(r_o, t)=0$. (Refer to the given Table 1) (7%)

10. Determine the integral $\int_0^\infty \frac{x\sin x}{(x^2+1)(x^4+4)}dx$ (8%)

Table 1: Bessel Function

x	$J_0(x)$	$J_1(x)$	$Y_0(x)$	$Y_1(x)$
0	1.0000	0.0000	—	—
1	0.7652	0.4401	0.0883	−0.7812
2	0.2239	0.5767	0.5104	−0.1070
3	−0.2601	0.3391	0.3769	0.3247
4	−0.3971	−0.0660	−0.0169	0.3979
5	−0.1776	−0.3276	−0.3085	0.1479
6	0.1506	−0.2767	−0.2882	−0.1750
7	0.3001	−0.0047	−0.0259	−0.3027
8	0.1717	0.2346	0.2235	−0.1581
9	−0.0903	0.2453	0.2499	0.1043
10	−0.2459	0.0435	0.0557	0.2490
11	−0.1712	−0.1768	−0.1688	0.1637
12	0.0477	−0.2234	−0.2252	−0.0571
13	0.2069	−0.0703	−0.0782	−0.2101
14	0.1711	0.1334	0.1272	−0.1666
15	−0.0142	0.2051	0.2055	0.0211

$J_0(x) = 0$ for x = 2.40483, 5.52008, 8.65373, 11.7915, 14.9309,
$J_1(x) = 0$ for x = 3.83171, 7.01559, 10.1735, 13.3237, 16.4706.

參考用

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