

1. (15%) Solve the following differential equation:

$$y'(\sinh 3y - 2xy) = y^2$$

2. (15%) Calculation of Convolution by integrating the following:

$$t * e^{at}$$

3. (15%) Find the orthogonal projection of $\begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$ onto $\text{Span}\left\{\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}\right\}$.

4. (15%) Compute $\det(B^2)$, where $B = A^{-1}$ and $A = \begin{bmatrix} 0 & 2 & 0 & -2 \\ 1 & 5 & 3 & -5 \\ 2 & -7 & 6 & 4 \\ -1 & 3 & 2 & -2 \end{bmatrix}$.

5. For two continuous-time signals $x(t)$ and $h(t)$, the $x(t)$ is an impulse train with inter-impulse interval T , and the $h(t)$ is a rectangular wave with unity gain from $t = 0$ to $t = T_0$ ($T_0 < T$). The mathematical representations of $x(t)$ and $h(t)$ are

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT), \text{ where } k \text{ belongs to integer, and } h(t) = u(t) - u(t - T_0), \text{ where } u(t)$$

is a unit step function with $u(t) = 1$ for $t \geq 0$. The convolution output $y(t)$ of these two signals is $y(t) = x(t) * h(t)$, in which the $*$ represents the continuous-time convolution operator. Please answer the following questions:

- (a) (2%) Is $y(t)$ a periodic signal or non-periodic signal?
 (b) (8%) Please determine the coefficients of Fourier series for $y(t)$, if $y(t)$ is a periodic signal. Otherwise, please determine the complex Fourier transform for $y(t)$, if $y(t)$ is a non-periodic signal.
6. (10%) Find the center and radius of convergence of the following power series:

$$\sum_{k=0}^{\infty} \left[1 + (-1)^k + \frac{1}{4^k} \right] z^k.$$

7. For the function $f(z) = \frac{(\sin z) \cdot (\cos 2z)}{z^3}$:

- (10%) (a) Find the first three non-zero terms of its Laurent expansion about $z = 0$.

- (10%) (b) Evaluate $\oint_C f(z) dz$, where C is the circle $|z - 1| = 2$.