科目：工程數學B(3004)　　校系所組：中央大學通訊工程學系（甲組）
中央大學電機工程學系(電子組)
交通大學電子研究所(甲組、乙A組、乙B組)
交通大學電控工程研究所(甲組、乙組)
交通大學電信工程研究所（甲組、乙B組）
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1. (12%) Which of the following statement(s) is(are) true:
   (a) Let $\boldsymbol{P}_2$ be the set of all polynomials of the form
   $$p(x) = a_2x^2 + a_1x + a_0,$$
   where a_0, a_1 and a_2 are real number. $\boldsymbol{P}_2$ is a vector space.
   (b) Let $\mathbf{W} = \{(x_1,x_2): x_1 \geqq 0 \text{ and } x_2 \geqq 0\}$, with the standard addition and scalar multiplication operations. $\mathbf{W}$ is a subspace of $\mathbf{R}^2$.
   (c) The set $\mathbf{S} = \{(1,2,3), (0,1,2), (-1,0,1)\}$ spans $\mathbf{R}^3$.
   (d) Let the matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 3 & 9 & 0 & -12 \end{bmatrix}$. The row space and column space of $\mathbf{A}$ have the same dimension.
   (e) Let the coordinate matrix of x in $\mathbf{R}^2$ relative to the ordered basis $\mathbf{B} = \{(1,0), (1,2)\}$ be $[x]_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. The coordinate matrix of x relative to the standard basis $\mathbf{B'} = \{(1,0), (0,1)\}$ is $[x]_{B'} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$.
   (f) The set $\{1, \cos x, \sin x\}$ is linearly dependent.

2. (8 %) Sketch the image of the triangle with vertices (0, 0), (1, 0), and (0, 1) under the linear transformation defined by the matrix product $\begin{bmatrix} 1 & 0 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ and give a geometric description.

3. (20 %) Let $U = Span\left\{\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}\right\}$ and $W = Span\left\{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\right\}$, two subspaces of $\mathbb{R}^3$.
   (a) (6 %) Find the orthogonal projection of $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ onto the subspace U.
   (b) (8 %) Find a matrix $\mathbf{P} \in \mathbb{R}^{3\times 3}$ such that $\mathbf{Pu} = \mathbf{u}$ for all $\mathbf{u} \in U$ and $\mathbf{Pw} = \mathbf{0}$ for all $\mathbf{w} \in W$ ($\mathbf{0}$ denotes the zero vector in $\mathbb{R}^3$).
   (c) (6 %) Following (b), find the eigenvalues and eigenvectors of $\mathbf{P}$.

注意：背面有試題

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4. (20 %) A system of linear equations has uncertain coefficients which are modeled by a random variable A. The system is as follows:

$$\begin{bmatrix} 1 & A & 0 \\ 0 & 1-A^2 & 1 \\ 0 & 0 & 1+A^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

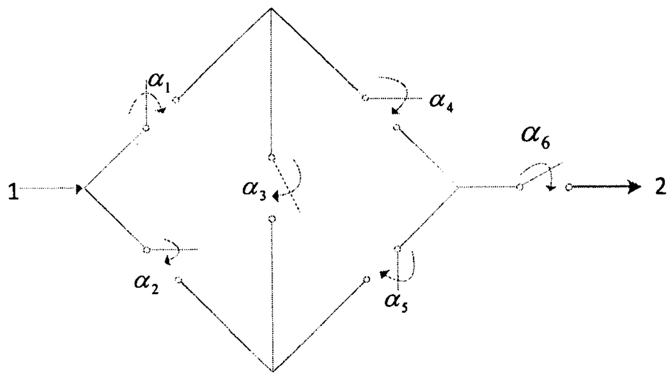
The probability mass function of A is

$$p_A(a)=\begin{cases} 0.2, & \text{if } a=1, \\ 0.3, & \text{if } a=0, \\ 0.5, & \text{if } a=-1. \end{cases}$$

(a) (10%) Please calculate the probability that the system has nontrivial solutions.

(b) (10%) Please solve the system when nontrivial solutions exist.

5. (10%) Switches $\alpha_1, \alpha_2, \cdots, \alpha_6$ in the diagram open and close randomly and independently. The probability that switch α_k is closed at any time equals p_k. Calculate the probability that at any time there is at least one closed path from point 1 to point 2.



6. (10%) The probability that a certain diode will fail before 1500 hours' service equals 0.4. If 10,000 such diodes are tested, use the central limit theorem to estimate the probability that between 3950 and 4180 will have failed before 1500 hours.

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7. (20%) A transmitter is sending a signal repeatedly to a receiver until the signal is received. Suppose that the signal is sent N times, where N is a geometric random variable with probability mass function (PMF) $p_N(n) = (1-p)^{n-1}p$ and p=0.8. Let the intensity of the received signal be X, where X is a normal random variable whose probability density function (PDF) conditioning on N is $f_{X|N}(x|n) = \frac{1}{\sqrt{2\pi n}} \exp\left\{-\frac{\left(x-\frac{n}{2}\right)^2}{2n}\right\}$.

(a) (10%) Find the unconditional mean and variance of X.

(b) (10%) Let $p_{N|X}(n|x)$ be the conditional PMF of N given X. Find the ratio

$$\frac{p_{N|X}(n=4|x=2)}{p_{N|X}(n=2|x=2)}.$$