

1. Reduce to first order and solve

$$x^2 y'' + xy' - 4y = 0, \quad y_1 = x^2$$

(15%)

2. Solve the following initial value problem by Laplace transform

$$y'' + 2y' + 5y = 50t - 100, \quad y(2) = -4, \quad y'(2) = 14$$

(15%)

3. Show orthogonality on the given interval.

$$1, x, x^2 - \frac{1}{3}, x^3 - \frac{3}{5}x \quad -1 \leq x \leq 1$$

(15%)

4. Solve the following problem by Laplace transform,

$$\frac{\partial u}{\partial x} + 2x \frac{\partial u}{\partial t} = 2x, \quad u(x, 0) = 1, \quad u(0, t) = 1.$$

(15%)

5. Find the corresponding Fourier series of the following periodic function,

$$f(x) = \pi x^3 / 2, \quad -1 < x < 1, \quad f(x+2) = f(x)$$

(15%)

6. Let  $\mathbf{A} = \begin{bmatrix} 2 & -1 & 3 \\ -2 & 1 & 4 \\ 1 & 2 & -2 \end{bmatrix}$ ,  $\mathbf{B} = \begin{bmatrix} -1 & 3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ ,  $\mathbf{a} = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}$ , and  $\mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ . Please

calculate  $\mathbf{AB}^T$ ,  $(3\mathbf{A} - 2\mathbf{B})^T \mathbf{a}$ ,  $\mathbf{a}^T \mathbf{A} \mathbf{a}$ ,  $\mathbf{a} \cdot \mathbf{b}$ ,  $\det(\mathbf{AB})$

(15%)

7. Let  $f = zy + yx$ ,  $\mathbf{v} = [y \ z \ 4z - x]$ ,  $\mathbf{w} = [y^2 \ z^2 \ x^2]$ . Find

- a.  $\nabla f$                       b.  $\nabla \cdot \mathbf{v}$                       c.  $\nabla \times \mathbf{w}$   
d.  $\nabla^2 f^2$                       e.  $\nabla \cdot (\mathbf{v} \times \mathbf{w})$

(10%)

參考用