科目 工程數學 類組別 031 共1頁 第1頁

＊請在試卷、答案卡內作答

參考用

1. (10%) Suppose we have a vector $\vec{F} = (x^2 + xy)\hat{e}_x + (xy + y^2)\hat{e}_y$, where $\hat{e}_x$ and $\hat{e}_y$ are unit vectors in the x- and the y- directions. Find $\nabla \times \vec{F}$.

2. (12%) A plane goes through three points, $A(1, 0, 0)$, $B(0, 1, 0)$, and $C(0, 0, 1)$, on the axes of the coordinate system. What is the unit vector of its surface normal $\hat{N}$? Find the directional derivative of a function $G(x, y, z) = x^2 y$ at the location $A(1, 0, 0)$ in the direction $\hat{N}$.

3. (12%) If matrix $\mathbf{H} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$, find its inverse matrix $\mathbf{H}^{-1}$.

4. (16%) Matrix **V** is the product of matrix **U** with matrix **U**, where matrix $\mathbf{U} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. Find the eigenvalues and the eigenvectors of the matrix **V**.

   [You may need to use $\sin(2\beta) = 2\sin\beta\cos\beta$ and $\cos(2\beta) = \cos^2\beta - \sin^2\beta$ to simplify your calculation. Be sure to obtain the answers in their simplest forms.]

5. (15%) Solve the initial value problem:

$$\frac{dy}{dt} + 2y = f(t),\ \ y(0) = 0 \ \text{ and } \ f(t) = \begin{cases} 1, & 0 \le t \le 1; \\ 0, & t > 1. \end{cases}$$

6. (13%) Consider an LRC circuit whose charge $q(t)$ is governed by the ODE $q'' + 40q' + 2000q = 520\sin(60t)$. Find the steady-state current.

7. (12%) Expand $f(x) = 2x^2 - 1$, $-1 < x < 1$, in a Fourier series.

8. (10%) The general solution of Bessel's equation, $x^2 y'' + xy' + (x^2 - \nu^2)y = 0$, is $y(x) = C_1 J_\nu(x) + C_2 J_{-\nu}(x)$ for $\nu \notin Z$, where $J_\nu(x)$ is the Bessel function of the first kind of order ν. Find the general solution of $x^2 y'' + xy' + (36x^2 - 2)y = 0$.