台灣聯合大學系統 94 學年度學士班轉學生考試命題紙

科目：工程數學　　類組別：D3. D4. D5　　共 1 頁第 1 頁　　*請在試卷答案卷（卡）內作答

1. (a) $y' + p(x)\,y = q(x)$. Find the general solution. Then use the result to solve $y' + 3\,y = x$, $y(a) = b$. (10%)

   (b) Solve the equation governing a mechanical oscillator: $mx'' + cx' + kx = F\cos\omega t$. (5%)

   (c) $x^2y'' - 2xy' - 10y = 0$, $y(a) = \alpha$, $y(b) = \beta$. (5%)

2. (a) $F(s) = \int_0^\infty f(t)e^{-st}\,dt$. If $f(t) = e^{at}$, calculate the corresponding $F(s)$. (5%)

   (b) For a general $f(t)$, obtain Laplace transforms for $f'(t), f''(t), f'''(t)$, and $f''''(t)$, supposing that the functions satisfy the requirement for existence of the Laplace transforms. (5%)

   (c) Using the above results to solve $y'''' - y = 0$, $y(0) = 1$, $y'(0) = y''(0) = y'''(0) = 0$. (5%)

3. (a) $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, obtain the eigenvalues and eigenspaces. (5%)

   (b) Solve $\begin{cases} x' = x + 4y \\ y' = x + y \end{cases}$, using the method of elimination to uncouple them. (5%)

   (c) Use a different approach that leads to an eigenvalue problem. Since the above equation is linear, constant-coefficient, and homogeneous, we can seek exponential solutions in the form: $x(t) = q_1e^{rt}$, $y(t) = q_2e^{rt}$. Substitute these into the equations and solve the eigenvalue problem. (5%)

4. (a) Evaluate $\int_{(-2,3,1)}^{(0,0,0)} \left[2xzdx + 2yzdy + (x^2 + y^2)dz\right]$. (5%)

   (b) Evaluate $\iint_R x^2\,dA$; R is the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. (5%)

   (c) Find $\nabla\cdot\left(\nabla\times\vec{F}\right)$ for the vector field $\vec{F} = x^2y\vec{i} + xy^2\vec{j} + 2xyz\vec{k}$. (5%)

5. (a) Find the Fourier series of $f(x)$, where $f(x) = x + \pi$, $-\pi < x < \pi$. (10%)

   (b) Use the result of (a) to find $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ (5%)

6. (a) Use separation of variables to find product solutions of $\frac{\partial^2 u}{\partial x \partial y} = u$. (5%)

   (b) Using the Laplace transform to solve a boundary-value problem:

   $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2},\ 0 < x < 2,\ t > 0,$$

   subject to $\begin{cases} u(0,t) = 0, u(2,t) = 0, t > 0 \\ u(x,0) = 0, \left.\frac{\partial u}{\partial t}\right|_{t=0} = \sin \pi x, 0 < x < 2 \end{cases}$ (15%)