Please show the details of your work for all questions. (two pages)

1. Test for exactness. If exact, solve. If not, find an integrating factor and then solve.

(a) $e^{-2\theta}\left(rdr - r^2 d\theta\right) = 0$ (10%)

(b) $\left(2\cos y + 4x^2\right)dx - x\sin y dy = 0$ (10%)

2. Solve the initial value problem.

$y'' + 4y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = -5$ (10%)

3. Solve the ordinary differential equation.

$y'' + 4y' + 4y = \cos 4t$ (10%)

4. Find the inverse Laplace transfrom of $f(t)$ of

$$F(s) = \frac{e^{-s}}{s^2 + \pi^2} + \frac{e^{-3s}}{(s+2)^2}$$ (10%)

5. Find the inverse of the matrix

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}.$$ (10%)

6. Find the Fourier series of the function $f(x)$, which is assumed to have the period 2π, and $f(x) = x^2 \ (-\pi < x < \pi)$. (10%)

注意：背面有試題

7. Using separating of varables to slove the following partial differential equations. (10%)

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2},\ u(0,t) = u(L,t) = 0,$$

$$u(x,0) = f(x),\ u_t(x,0) = 0,\ (0 \le x \le L).$$

8. Find curl $\vec{v}$, where $\vec{v} = \left[\ln(x^2+y^2),\ 2\tan^{-1}\left(\frac{y}{x}\right),\ 0\right]$ is given with respect to right-handed Cartesian coordinates. (10%)

9. $\int_{-L}^{L} \cos\frac{m\pi x}{L}\cos\frac{n\pi x}{L}\,dx = ?$ [m, n are integers.] (10%)