科目 微積分 類組別 A2,A3,A4,A5,A6,A10 共 1 頁第 1 頁 *請在試卷答案卷（卡）內作答

甲、填充題：共 4 大題，每大題 10 分，共 40 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Let

$$f(x) = \begin{cases} \frac{9x-3\sin 3x}{5x^3}, & x \neq 0 \\ a, & x = 0. \end{cases}$$

If the function f is continuous on the entire real line. Then $a =$ ______.

2. Let

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Then $f_{xy}(0,0) =$ ______ and $f_{yx}(0,0) =$ ______.

3. Let $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$. Then the interval of convergence of $f(x)$ is ______, and the interval of convergence of $f'(x)$ is ______.

4. Let R be the region inside the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ where $a, b, c > 0$, and above the plane $z = b - y$, then the volume of the region R is ______.

乙、計算、證明題：共 6 大題，每大題 10 分，共 60 分。須詳細寫出演算過程、否則不予計分。

5. Test the series for convergence or divergence using any appropriate test. Identify the test used and explain your reasoning.

(a) $\displaystyle\sum_{n=1}^{\infty} n2^{-n} \sin \frac{1}{n}$. (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{2^{n+1}+1}$.

6. Find the absolute maximum and minimum values of

$$f(x,y,z) = 2x - 2y + z$$

subject to the constraint $x^2 + y^2 + z^2 = 1$.

7. Evaluate the following integrals.

(a) $\displaystyle\int \sin(\ln x)\,dx$. (b) $\displaystyle\int_0^1 \frac{x^3-1}{\ln x}\,dx$. **Hint:** $\displaystyle\int_0^3 a^x\,dx = \frac{a^3-1}{\ln a}$ for $a > 0, a \neq 1$.

8. Find the following limits.

(a) $\displaystyle\lim_{x\to 0^+} x \int_x^1 \frac{\cos t}{t^2}\,dt$. (b) $\displaystyle\lim_{x\to 0} \frac{x^2 \sin \frac{1}{x}}{\tan x}$.

9. Show that

$$0 < \int_{-\frac{1}{\sqrt{2}}}^{1} \frac{\sin(\pi x^2)}{x}\,dx < \frac{1}{2}\ln 2.$$

10 Consider the function $f(x,y) = \alpha x^2 + \beta y^2$. Find values for α and β so that (a) (0,0) is a local minimum (b) (0,0) is a local maximum, and (c) (0,0) is a saddle.