

科目 微積分

類組別 A2, A3, A4

A5, A6, B6, B7 共 2 頁第 1 頁

\*請在試卷答案卷(米)內作答

一. 填充題： 請將答案按字母順序寫在答案紙前八行。 不要寫計算過程。 (每格 8 分)

1.  $\int_0^{\pi} x \sin x dx =$  A .

2. For the equation  $y^3 - xy^2 + \cos(xy) = 2$ ,  $\frac{dy}{dx}$  at the point (0,1) is B .

3. Let  $F(t) = \int_{g(t)}^{h(t)} f(u) du$ , where  $f$  is continuous and  $g$  and  $h$  are differentiable. Find  $F'(t) =$  C .

4. The maximum value of  $f(x, y) = y^2 - x^2$  on the ellipse  $\frac{x^2}{4} + y^2 = 1$  is D .

5.  $\int_0^1 \int_x^1 e^{y^2} dy dx =$  E .

6.  $\lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right) =$  F .

7. Let  $f: [0, 3] \rightarrow [0, 3]$  be a continuous function with  $f(0) = 0$  and  $f(3) = 3$ . If  $f$  is one-to-one and  $\int_0^3 f(x) dx = \frac{9}{5}$ , then  $\int_0^3 f^{-1}(x) dx =$  G , where  $f^{-1}$  is the inverse function of  $f$  .

8. The radius of convergence of the power series  $\sum_{k=1}^{\infty} \left( \frac{k+1}{k} \right)^{k^2} x^k$  is H .

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類組別  $A_2, A_3, A_4,$   
 $A_5, A_6, B_1, B_2$  共 2 頁第 2 頁 \*請在試卷答案卷(卡)內作答

二. 計算與證明：請詳細寫出每一個推導步驟。(每題 12 分)

1. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be twice differentiable. If  $f''$  is nowhere vanishing, then  $f$  has at most two distinct real roots.

2. Prove that  $\int_0^1 e^{-t^2} dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)2^k k!}$ .

3. Evaluate  $\oint_C (x^3 + y^3)dx + (2y^3 - x^3)dy$ , where  $C$  is the unit circle.