

一、(20%) Please solve the following ordinary differential equation (ODE):

$$y'(x) = 3y(x), \text{ with } y(0) = 3.$$

二、(20%) Please solve the following ordinary differential equation (ODE):

$$5x^2 - 3y^2 + 6xyy' = 0, \text{ with } y(1) = 2.$$

三、(20%) Please use Laplace transform to solve the following ordinary differential equation (ODE):

$$y''(t) + 5y'(t) + 6y(t) = u(t-2), \text{ with } y(0) = 0 \text{ and } y'(0) = 0.$$

四、(20%) Please find the inverse Fourier transform of the following time domain signals:

$$(一) (10\%) \quad X(j\omega) = \frac{3j\omega + 24}{(j\omega)^2 + 5j\omega + 4}.$$

$$(二) (10\%) \quad X(j\omega) = \begin{cases} 1, & -10 < \omega < 10 \\ 0, & \text{otherwise} \end{cases}.$$

五、(20%) Please prove the following relations:

$$(一) (10\%) \quad F\{x(t) * g(t)\} = X(j\omega) \cdot G(j\omega),$$

where $F\{\cdot\}$ is the Fourier transform operator, $*$ is the continuous-time convolution operator, and $X(j\omega)$ and $G(j\omega)$ are the Fourier transforms of $x(t)$ and $g(t)$, respectively.

$$(二) (10\%) \quad L\{f(t) * g(t)\} = L\{g(t) * f(t)\},$$

where $L\{\cdot\}$ is the Laplace transform operator, $*$ is the continuous-time convolution operator, and $f(t)$ and $g(t)$ are two continuous-time signals.

