

甲、填充題：共 10 題，每題 8 分，共 80 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

- Find the value of  $a$  such that  $\lim_{x \rightarrow 0} \frac{\sqrt[3]{ax+8} - 2}{x} = \frac{5}{12}$ . Answer : \_\_\_\_\_
- The coordinates of a particle in the  $xy$ -plane are differentiable functions of time  $t$  with  $dx/dt = -1$  m/sec and  $dy/dt = -5$  m/sec. How fast is particle's distance from the origin changing as it passes through the point  $(5, 12)$ ? Answer : \_\_\_\_\_
- What is the value of the definite integral  $\int_0^1 \frac{x}{\sqrt{4+5x}} dx$ ? Answer : \_\_\_\_\_
- Find  $f(4)$  if  $\int_0^{x^2} f(t) dt = x \cos \pi x$ . Answer : \_\_\_\_\_
- Find the length of the curve  $y = \int_0^x \sqrt{\cos 2t} dt$   $0 \leq x \leq \pi/4$ . Answer : \_\_\_\_\_
- Evaluate  $\int_1^\infty \frac{\ln x}{x^2} dx$ . Answer : \_\_\_\_\_
- Find a value for the constant  $b$  that will make the radius of convergence of the power series  $\sum_{n=2}^\infty \frac{b^n x^n}{\ln n}$  equal to 5. Answer : \_\_\_\_\_
- Find the value of  $a$  that make  $f(x, y) = \begin{cases} \frac{\sin xy}{xy}, & \text{if } xy \neq 0 \\ a, & \text{if } xy = 0. \end{cases}$  continuous on  $\mathbb{R}^2$   
Answer : \_\_\_\_\_
- Find the area of the region  $R$  in the  $xy$ -plane enclosed by the circle  $x^2 + y^2 + 4$ , above the line  $y = 1$ , and below the line  $y = \sqrt{3}x$ . Answer : \_\_\_\_\_
- Let  $f(x, y) = x^2 - xy + y^2 - y$ . Find the direction  $u$  for which  $D_u f(1, -1) = 4$ .  
Answer : \_\_\_\_\_

乙、計算、證明題：共 2 題，每題 10 分，共 20 分。須詳細寫出計算及證明過程，否則不予計分。

- Find the highest and lowest points on the curve  $x^2 + xy + y^2 = 12$ .
- (a) If  $\mathbf{F}(x, y) = x^2 \mathbf{i} + y^2 \mathbf{j}$ , find a function  $f$  such that  $\mathbf{F} = \nabla f$ .  
(b) Evaluate the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$ , where  $C$  is the arc of the parabola  $y = 2x^2$  from  $(-1, 2)$  to  $(2, 8)$ .

