

國立中央大學104學年度碩士班考試入學試題

所別：電機工程學系碩士班 固態組(一般生) 科目：工程數學 共 1 頁 第 1 頁

電機工程學系碩士班 系統與生醫組(一般生)

本科考試禁用計算器

*請在答案卷(卡)內作答

參考用

一、(10%) Please answer the following questions

(一) (3%) If $D=P^{-1}AP$ and D is diagonal, prove that $A^k=PD^kP^{-1}$.

(二) (7%) Find A^{58} , where $A = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$.

二、(10%) Prove that: If $\{v_1, v_2, \dots, v_n\}$ is an orthogonal set of nonzero vectors in an inner product space V , then v_1, v_2, \dots, v_n are linearly independent.

三、(10%) Find the rank and nullity of the matrix A , where

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 3 & 9 & 0 & -12 \end{bmatrix}$$

四、(15%) Find all roots of the equation $\sin(z) = \cosh(4)$.

五、(15%) Evaluate the value of the integral $\int_{-\infty}^{\infty} \frac{x^3 \sin(ax)}{x^4+4} dx$ ($a > 0$).

六、(15%) Find the response of the following undamped mass-spring system.

$$y'' - y = 10\delta(t - \frac{1}{2}) - 100\delta(t - 1), \quad y(0) = 10, \quad y'(0) = 1$$

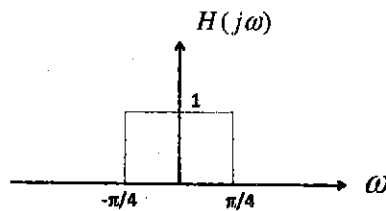
七、(15%) Find the solution of the following equation. (Please show the details of your solution)

$$y(t) * t^{-1/2} = 1 + t,$$

where $*$ is the continuous-time convolution operator.

(hint: Gamma function $\Gamma(a) = \int_0^{\infty} e^{-t} t^{a-1} dt$, $\Gamma(1/2) = \sqrt{\pi}$)

八、(10%) For a continuous-time low-pass filter system with its Fourier response $H(j\omega)$



Please answer the following questions:

(一) (7%) Please use continuous-time inverse Fourier transform to find the impulse response $h(t)$ of the low-pass filter system.

(二) (3%) For an input signal $x(t) = \sin(\frac{\pi t}{6}) + \cos(\frac{\pi t}{2})$, please find the output of this low-pass filter.