國立中央大學八十三學年度研究所碩士班入學試題卷

系所別： 電機工程研究所 組 科目： 工程數學 共 1 頁 第 1 頁

1. (10%) Solve the differential equation: $(x^2 - x)y'' - xy' + y = 0$.

2. (10%) Suppose that $f(t)$ is continuous, except for an ordinary discontinuity (finite jump) at $t = a (> 0)$, and we have $\lim_{\epsilon\to 0} f(a+\epsilon) = f(a+0)$ and $\lim_{\epsilon\to 0} f(a-\epsilon) = f(a-0), \epsilon > 0$. Furthermore, $f(t)$ satisfies

$$|f(t)| \leq Me^{\gamma t}, \text{ for all } t \geq 0.$$

for some M and γ, and has a derivative $f'(t)$ that is piecewise continuous on every finite interval in the range $t \geq 0$. If $F(s)$ represents the Laplace transform of $f(t)$, that is, $F(s) = L(f(t))$. Find the Laplace transform of the derivative $f'(t)$, where $f'(t) = \frac{d}{dt}f(t)$.

3. (10%) Find out what type of conic section the following quadratic form represents and transform it to principal axes. Express $x^T = [x_1 x_2]$ in terms of the new coordinate vector $y^T = [y_1 y_2]$.

$$Q = 17x_1^2 - 30x_1x_2 + 17x_2^2 = 128.$$

4. (10%) Show that a subset of a linearly independent set is itself linearly independent.

5. (10%) Integrate

$$I = \int_c [2xyz^2dx + (x^2z^2 + z\cos yz)dy + (2x^2yz + y\cos yz)dz]$$

along the straight line from A:$(0,0,1)$ to B:$(1, \frac{\pi}{4}, 2)$.

6. (10%) Find the Fourier cosine integral of:

$$f(x) = e^{-kx}, (x > 0, k > 0).$$

7. (10%) Evaluate the following integral where C is the unit circle (counterclockwise).

$$\oint_C e^{\frac{1}{z}} dz.$$

8. (10%) Evaluate the integral:

$$\int_0^\infty \frac{\cos 2x}{4x^4 + 13x^2 + 9} dx.$$

9. (10%) Find a linear fractional transformation that maps $0, -i, i$ onto $-1, 0, \infty$, respectively.

10. (10%) Solve the problem:

$$\frac{\partial w}{\partial x} + x\frac{\partial w}{\partial t} = 0, w(x,0) = 0, w(0,t) = t, (t \geq 0).$$