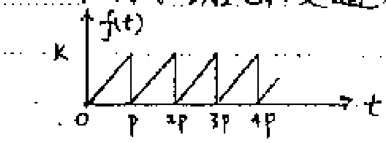
1. 求解微分方程式 $y' + \frac{1}{x} y = x y^2$ (10%)

2. 求解始值微方 $y'' - 2y' + y = e^x + x$ (10%)

$y(0) = 1$, $y'(0) = 0$

3. 求圖示鋸齒波週期函數的拉氏轉換 (15%)



$f(t) = \frac{k}{p} t$ 若 $0 < t < p$.

$f(t+p) = f(t)$.

(4) (a) Please draw the equation $f(x, y, z) = 0$ in $x - y - z$ 3-dimension space, where $f(x, y, z) = 4(x^2 + y^2) - z^2$. (5%)

(b) Find the gradient of $f(x, y, z)$ at a point P_0, that is, $\nabla f(P_0)$, where $P_0 = (1, 0, 2)$; and explain what is the mathematic meaning of $\nabla f(P_0)$? (5%)

(c) Find a unit normal vector N of the surface $f(x, y, z) = 0$ at P_0. (5%)

(d) Find the "Directional Derivative" of f at P_0 in the direction of N. (5%).

(5) (a) If A is partitioned as
$$A = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}$$
where A_{11} and A_{22} are two square matrices, show that $det(A) = det(A_{11})det(A_{22})$, where $det(A)$ denotes the "Determinant" of the matrix A. (5%)

(b) Find the characteristic polynomial of
$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -6 & 5 \end{bmatrix}$$
and its eigenvalues; by the way, is $A^2 - 5A + 6I = 0$? where I is an Identity matrix. If yes, give the proof; if not, why? (5%)

15% 6. The bilateral Laplace transform of x(t) is defined as $X(s)=\int_{-\infty}^{\infty} x(t)e^{-st}dt$ and the inverse Laplace transform is given by $x(t)=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} X(s)e^{st}ds$, where the contour of integration is a straight line parallel to the iw-axis in the complex s plane and is determined by any value of σ so that X(σ+iw) converges.

Use the residue theorem to find the inverse Laplace transform x(t) of $X(s)=\frac{1}{(s+1)(s+2)}$ with the region of convergence (ROC) given as follows:

(i) Re{s} < -2, (ii) Re{s} > -1, and (iii) -2 < Re{s} < -1.

Note that Re{s} represents the real part of s.

20% 7. The bilateral z-transform of a discrete sequence x[n] is defined as $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}$ and its inverse z-transform is given by $x[n]=\frac{1}{2\pi i}\oint X(z)z^{n-1}dz$, where the contour is a counterclockwise closed circular contour in the complex z plane, centered at the origin and with radius r which can be chosen as any value for which X(z) converges.

Use the residue theorem to find the inverse z-transform x[n] of $X(z)=\frac{1}{1-\frac{1}{4}z^{-1}}$ with the region of convergence (ROC) given as follows: (i) |z| >1/4, and (ii) |z| < 1/4.

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