1.(10%) Twelve different message signals, each with a bandwidth of 10 kHz, are to be multiplexed and transmitted. Determine the minimum bandwidth required for each method if the multiplexing or modulation method used is (a) FDM (b) SSB (c) PAM (d) TDM

2.(10%) Determine the range of permissible cutoff frequencies for the ideal lowpass filter used to reconstruct the signal $x(t)=10\cos(600\pi t)\cos^2(1600\pi t)$ which is sampled at 4000 samples per second. Sketch X(f) and the spectrum of sampled signal, $X_\delta(f)$. Find the minimum allowable sampling frequency.

3. (10%) The random variable x has a normal distribution $f_x(x)=(2\pi\sigma^2)^{-1/2}\exp(\frac{-x^2}{2\sigma^2})$.

Find the probability density function $f_Y(y)$ of the random variable $y=\begin{Bmatrix} x^2, x\ge 0, \\ 0, \ x<0. \end{Bmatrix}$

4. (10%) The x and y are independent and Gaussian with expected values A and 0, respectively, and equal variaces σ^2. Find the probability density function $f_z(z)$ of $z=(x^2+y^2)^{1/2}$ in terms of the modified Bessel function $I_o(x)=(2\pi)^{-1}\int_0^{2\pi}\exp(x\cos\theta)d\theta$.

5. (10%) The conditional probability density functions corresponding to two hypotheses (H_1 versus H_o) are

$$f_{Y_i|H_1}(y|H_1)=\frac{1}{\beta}e^{-y/\beta}u(y)$$

$$f_{Y_i|H_0}(y|H_0)=\frac{1}{\alpha}e^{-y/\alpha}u(y)$$

with $\beta>\alpha>0$ and $u(y)$ being the unit step function. Suppose we want to test these hypotheses based on two independent samples Y_1 and Y_2.

(a) Derive the maximum a *posteriori* (MAP) decision rule for the test.

(b) Calculate the detection and false alarm probabilities if $\alpha=2$, $\beta=4$, and $P_r\{H_o\}=P_r\{H_1\}=\frac{1}{2}$.

6. (10%) Consider the signal $r(t)=A[1+m(t)]\cos(2\pi f t+\theta)+n_c(t)\cos(2\pi f t+\theta)-n_s(t)\sin(2\pi f t+\theta)$.

This signal can also be expressed in the polar form $r(t)=R(t)\cos(2\pi f t+\theta+\phi(t))$

where $R(t)$ and $\phi(t)$ are the corresponding envelope and phase. Find $R(t)$ and $\phi(t)$ in terms of A, $m(t)$, $n_c(t)$ and $n_s(t)$.

7. (10%) Consider phase-shift keying with a carrier component which can be written as

$x_c(t)=A_c\sin[2\pi f_c t+d(t)\cos^{-1}a]$ where d(t) is binary data which is ±1-valued in contiguous T_b-second bit intervals, A_c is the amplitude. Find the powers, in terms of a and A_c in carrier and modulation components.

8. (10%) An MSK system has a carrier frequency of 1 Mhz and transmits data at a rate of 100 kbps.

(a) Find the tone frequency ?

(b) For the data sequence 000000000...., what is the instantaneous frequency?

9. (10%) Consider A(7,4) systematic block code has parity-check equation

$P_1 = m_1 + m_2 + m_4$

$P_2 = m_2 + m_3 + m_4$

$P_3 = m_1 + m_3 + m_4$

where m_i are message digits and P_i are check digits.

(a) Find the generator matrix.

(b) Find the parity-check matrix.

(c) Encode the message 1011.

(d) Decode the sequences 1100111 and find the syndrome.

10. (10%) Consider five messages given by the probabilities 1/2, 1/4, 1/8, 1/16, 1/16.

(a) Calculate the average information content.

(b) Use the Shannon-Fano algorithm to develop an efficient code and, for that code, calculate the average number of bits/message. Compare with H.