

系所別: 電機工程學系 乙組 科目: 近代物理

1. A particle of mass  $m$  moves in a two-dimensional box of sides  $L$ . (a) Write expressions for the wavefunctions and energies as a function of the quantum numbers  $n_1$  and  $n_2$ . (b) Find the energies of the ground state and first excited state. Is either of these states degenerate? Explain. (10 points)
2. Consider a particle in an infinite square well described initially by a wave that is a superposition of the ground state and first excited state of the well:  
$$\Psi(x,0) = C[\Psi_1(x) + \Psi_2(x)]$$
(a) Show that the value  $C = 2^{-1/2}$  normalizes this wave, assuming  $\Psi_1(x)$  and  $\Psi_2(x)$  are normalized. (b) Find  $\Psi_1(x,t)$  at any later time  $t$ . (c) Show the superposition is not a stationary state, but that the average energy in this state is the arithmetic mean  $(E_1 + E_2)/2$  of the ground and first excited state energies of  $E_1$  and  $E_2$ . (15 points)
3. (a) Consider a system of electrons confined to a three-dimensional box. Calculate the ratio of the number of allowed energy levels at 8.5 eV to the number of allowed energy levels at 7.0 eV. (b) Copper has a Fermi energy of 7.0 eV at 300 K. Calculate the ratio of the number of occupied levels at an energy of 8.5 eV to the number of occupied levels at the Fermi energy. Compare the result with that obtained in part (a). (15 points)
4. Explain how the energy bands of metals, semiconductors, and insulators account for the following general optical properties. (a) Metals are opaque to visible light. (b) Semiconductors, such as Si and GaAs, are opaque to visible light but transparent to infrared. (c) Many insulators, such as diamond, are transparent to visible light. (15 points)
5. Explain the electron diffraction patterns of (a) amorphous, (b) polycrystalline, and (c) single crystalline Si schematically. (15 points)
6. Discuss the basic assumptions of Maxwell-Boltzmann, Fermi-Dirac, and Bose-Einstein statistics. How do they differ, and what are their similarities? (15 points)
7. Derive the density of state of electrons in metal and draw the curve showing electron distribution against energy at 0 K and room temperature. (15 points)

(Boltzmann's constant  $k_B = 1.381 \times 10^{-23}$  J/K, Planck's constant  $h = 6.625 \times 10^{-34}$  J.s)

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