國立中央大學97學年度碩士班考試入學試題卷

所別：電機工程學系碩士班 電波組　科目：工程數學　共 1 頁 第 1 頁

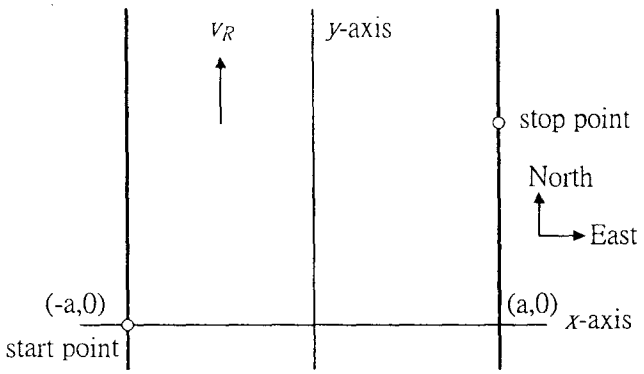
＊請在試卷答案卷（卡）內作答

參考用

1. (20%) The following figure shows a northward-flowing river of width $w = 2a$. The lines $x = \pm a$ represent the banks of the river, and the y-axis is its center. Suppose that the velocity with which the water flows increases as one approaches the center of the river, and indeed is given in terms of distance x from the center by

$$v_R = v_0\left(1 - \frac{x^2}{a^2}\right)$$

Suppose that a swimmer starts at the points $(-a, 0)$ on the west bank and swim due east (relative to the water) with a constant speed v_s. Determine the coordinate of the stop point given the river midstream velocity $v_0 = 9$ km/hour, the swimmer's velocity $v_s = 3$ km/hour, and the river width $w = 2a = 1$ km.



2. (20%) Solve the boundary value problem

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0$$
$$u(0,y) = u(a,y) = u(x,b) = 0.$$
$$u(x,0) = f(x)$$

3. (20%) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, where $a, b \in$ real constants.

4. (20%) Determine a such that $u = \sin ax \cosh 2y$ is harmonic, and find the harmonic conjugate of u.

5. (20%) Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{k + \cos\theta}$ $(k > 1)$.