所別： 資工類

科目： 離散數學與線性代數

本科考試禁用計算器 ＊請在答案卡內作答

參考用

單選題 每題五分

1. Which one of these is a proposition?
   (a) You shall not pass! (Curtesy of The Lord of the Rings)
   (b) What time is it?
   (c) I have a pen, I have a apple. Uh! Apple-Pen!
   (d) 4 + x = 5.
   (e) The moon is made of green cheese.
2. Which of the following statement about a bijection function is incorrect?
   (a) If $f: A \to B$ is injective, then the inverse function, f^{-1}, always exists and $f \circ f^{-1} = f^{-1} \circ f = I$.
   (b) If f is bijective, then f is both injective and surjective.
   (c) If a function is strictly increasing or strictly decreasing, it is a bijection.
   (d) An identity function is bijective.
   (e) No matter two sets A and B are finite, countably infinite, or uncountable, A and B are said to be of the same size if and only if there is a bijective function $f: A \to B$.
3. Given two arbitrary uncountable sets A and B, the set A − B is
   (a) finite.
   (b) countably infinite.
   (c) uncountable.
   (d) All of the above are possible.
   (e) Not all of the above are correct.
4. What is the next largest 4-combination of {1, 2, 3, 4, 5, 6} after {1, 2, 5, 6}?
   (a) {1, 3, 4, 5}.
   (b) {2, 3, 4, 5}.
   (c) {1, 3, 5, 6}.
   (d) {1, 2, 6, 6}.
   (e) None of the above.
5. The quick multiplication algorithm using the idea of divide-and-conquer can compute the product $c \cdot d$ of two $2n$-digit base-b numbers in time complexity
   (a) $\Theta(n)$.
   (b) $\Theta(n^{\log_2 3})$.
   (c) $\Theta(n^{\log_B 2})$.
   (d) $\Theta(n \log_2 n)$.
   (e) $\Theta(n^2)$.

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6. Let $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ be the matrix to represent a binary relation **R** on a four-elements set. Which of the following statements is true?

(a) The reflexive closure of **R** is an equivalence relation.

(b) There are eight 1's in the matrix that represents the symmetric closure of **R.**

(c) The directed graph representation of **R** is connected.

(d) The directed graph of **R** does not have a strongly connected component..

(e) None of the above.

7. Suppose x and y are integer numbers, and we define the following predicates:

$D(x,y)$: y is a multiple of x; $E(x)$: x is even;

Which of the following clauses are correct interpretations of the logical statement:

$\forall x, y, (E(x) \wedge (\neg E(y)) \rightarrow ((\neg D(x,y)) \wedge (\neg D(y,x)))$

(a) All even integers can only be a multiple of another non-even integer.

(b) If y is a multiple of x, it is not possible that x is odd and y is not even. .

(c) It is possible that some not-even number is not a multiple of some odd number..

(d) Some odd integer is not equal to an even integer.

(e) None of the above.

8. We implement a merge sort algorithm to sort $\boldsymbol{n}$ items. The algorithm will divide the set into 2 roughly equal-size halves, and merge the 2 halves after each half set is recursively sorted. Because the item comparison is complicated, the merge process takes $\theta(m\sqrt{m})$ steps for input size $\boldsymbol{m}$. What is the time complexity for this algorithm?

(a) $\theta(n \log n)$　(b) $\theta(n)$　(c) $\theta(n^2)$　(d) $\theta(n\sqrt{n})$　(e) $\theta(n\sqrt{n}\log n)$

9. If number of nodes is more than 3, which statement about graph is not true?

(a) Complete graphs have Hamilton circuit.

(b) Complete bipartite graphs have more edges than nodes.

(c) Any strongly connected directed graph has circuit.

(d) There is a length-n path between any 2 nodes in n-dimension hypercube.

(e) All connected undirected graph have a subgraph as a tree.

10. About formal proof, which following statements is true?

(a) If predicates form a partial order set, they can be proved by mathematic induction.

(b) Diagonalization proof can only be applied on finite sets.

(c) Two sets are equal if and only if they are mutual subsets.

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(d) Equal number of degree-n nodes is sufficient to prove graph isomorphism.

(e) Halting problem is a contradiction proof of the existence of intractable problems.

多重選擇題 (每一選項答對給 1 分、答錯扣 1 分、不答 0 分)

11. Which of the following vectors compose a basis for the column space of the matrix A?

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ 0 \\ 1 \\ 6 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 6 \\ -1 \\ 1 \\ 3 \end{bmatrix}$

12. We have five lists of polynomials in $P_3(R)$. Select the list in which the first polynomial can be expressed as a linear combination of the other two.

(a) $x^3 - 3x + 5, x^3 + 2x^2 - x + 1, x^3 + 3x^2 - 1$

(b) $4x^3 + 2x - 6, x^3 - 2x^2 + 4x + 1, 3x^3 - 6x^2 + x + 4$

(c) $-2x^3 - 11x^2 + 3x + 2, x^3 - 2x^2 + 3x - 1, 2x^3 + x^2 + 3x - 2$

(d) $x^3 + x^2 + 2x + 13, 2x^3 - 3x^2 + 4x + 1, x^3 - x^2 + 2x + 3$

(e) $x^3 - 8x^2 + 4x, x^3 - 2x^2 + 3x - 1, x^3 - 2x + 3$

13. Determine which of the following systems of linear equations has a solution

(a) $\begin{cases} x_1 + x_2 - x_3 + 2x_4 = 2 \\ x_1 + x_2 + 2x_3 = 1 \\ 2x_1 + 2x_2 + x_3 + 2x_4 = 4 \end{cases}$

(b) $\begin{cases} x_1 + x_2 - x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 2 \end{cases}$

(c) $\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ x_1 + x_2 - x_3 = 0 \\ x_1 + 2x_2 + x_3 = 3 \end{cases}$

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(d) $\begin{cases} x_1 + x_2 + 3x_3 - x_4 = 0 \\ x_1 + x_2 + x_3 + x_4 = 1 \\ x_1 - 2x_2 + x_3 - x_4 = 1 \\ 4x_1 + x_2 + 8x_3 - x_4 = 0 \end{cases}$

(e) $\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + x_2 + 2x_3 = 3 \\ x_1 - 4x_2 + 7x_3 = 4 \end{cases}$

14. Which of the following statements are true?
   (a) If B is a matrix obtained by interchanging two rows or two columns of A, then det(B)=det(A).
   (b) If B is a matrix obtained by multiplying each entry of some row or column of A by a scalar, then det(B)=det(A).
   (c) If B is a matrix obtained from A by adding a multiple of some row to a different row (or a multiple of some column to a different column), then det(B)=det(A).
   (d) The determinant of an upper triangular nxn matrix is the product of its diagonal entries.
   (e) If Q is an invertible matrix, then $\det(Q^{-1})=[\det(Q)]^{-1}$

15. Let β and γ be the standard ordered bases for Rn and Rm, respectively. For the following transformations: $T:R^n \rightarrow R^m$, choose the option in which $[T]_\beta^\gamma$ is correct.

   (a) $T:R^2 \rightarrow R^3$ defined by $T(a_1, a_2)=(2a_1-a_2, 3a_1+4a_2, a_1)$, $[T]_\beta^\gamma = \begin{pmatrix} 2 & -1 \\ 3 & 2 \\ 1 & 0 \end{pmatrix}$

   (b) $T:R^3 \rightarrow R^2$ defined by $T(a_1, a_2, a_3)=(2a_1+3a_2-a_3, a_1+a_3)$, $[T]_\beta^\gamma = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}$

   (c) $T:R^3 \rightarrow R$ defined by $T(a_1, a_2, a_3)=2a_1+a_2-3a_3$, $[T]_\beta^\gamma =(2, 1, -3)$

   (d) $T:R^3 \rightarrow R^3$ defined by $T(a_1, a_2, a_3)=(2a_2+a_3, -a_1+4a_2+5a_3, a_1+a_3)$,

   $$[T]_\beta^\gamma = \begin{pmatrix} 0 & 2 & 1 \\ -1 & 3 & 2 \\ 1 & 0 & 1 \end{pmatrix}$$

$T:R^n \rightarrow R^n$ defined by $T(a_1, a_2, \ldots, a_n)=(a_1, a_1, \ldots, a_1)$, $[T]_\beta^\gamma = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \end{pmatrix}_{n\times n}$

16.If A is an invertible and diagonalizable $n \times n$ matrix, then

(A) A and A^T have the same eigenvalues.
(B) A and A^T have the same eigenvectors.
(C) A and A^{-1} have the same eigenvectors.
(D) A^T is diagonalizable.
(E) A^{-1} is diagonalizable.

17.If A is a $n \times n$ matrix with real entries, then

(A) A has n eigenvalues exactly.
(B) A has n eigenvectors exactly.
(C) A has n eigenspaces exactly.
(D) The bases of all A's eigenspaces are not always linearly independent.
(E) If A has complex eigenvalues, then A must has complex eigenvectors.

18.If A can be QR factorized (*i.e.*, $A = QR$), then

(A) A has linearly independent eigenvectors.
(B) The columns of Q are orthonormal basis for Col A.
(C) $Q^T Q = I$ (identity matrix).
(D) $A = QR$ and $Q^T Q = I \Rightarrow Q^T A = Q^T QR \Rightarrow Q^T A = R \Rightarrow QQ^T A = QR = A \Rightarrow QQ^T = I$.
(E) R is an upper triangular invertible matrix with positive entries on its upper triangle.

19.If A is a $m \times n$ matrix; $Ax = b$ is an inconsistent system and has a least-square solution $\hat{x}$.

(A) $\hat{x} = (A^T A)^{-1} A^T b$.
(B) $\hat{x}$ is a unique solution.
(C) If $A^T A$ is not invertible, then the system has no solution.
(D) $A\hat{x}$ is not always in Col A.
(E) $b - A\hat{x}$ is orthogonal to every row of A.

20.If C is a covariance matrix, then

(A) C is a symmetric matrix.
(B) C has real entries and then has real eigenvalues.
(C) C has no negative eigenvalues.
(D) C can always be decomposed into AA^T, where A is a matrix.
(E) C can always be decomposed into PDP^T, where D is a diagonal matrix.