DISCRETE MATHEMATICS

1. (20%) Let $A = \{1, 2, 3, 4, 5\}$ and R be the relation defined on A by aRb if and only if $a < b$.

   (a) Compute R^2 and R^3.

   (b) Complete the following statement: aR^2b if and only if _____.

   (c) Complete the following statement: aR^3b if and only if _____.

2. (20%) Twenty cards numbered 1 through 20 are placed face down on a table. Cards are randomly selected by the player one at a time and turned over until 10 cards have been chosen. If two of the chosen cards add up to 21, the player loses.

   (a) Is it possible that the player wins this game? Explain your answer.

   (b) What is tha probability that the player wins this game?

3. (20%) How many 4-digit numbers $x_1x_2x_3x_4$ (can be selected) from 0 = 0000 to 9999 satisfying $x_i \leq x_{i+1}$ for i = 1, 2, and 3.

4. (20%) A connected graph G has 11 vertices and 53 edges. Show that G is Hamiltonian but not Eulerian.

5. (20%) State the language generated by the following context-free grammar:

$$S \longrightarrow AB \mid BA$$
$$A \longrightarrow CAC \mid a$$
$$B \longrightarrow CBC \mid b$$
$$C \longrightarrow a \mid b$$