所別：通訊工程學系碩士班 甲(通訊系統及訊號處理)組(一般生)　科目：工程數學(線性代數、機率)　共 2 頁 第 1 頁
通訊工程學系碩士班 乙(通訊網路)組(一般生)
本科考試禁用計算器　　*請在試卷答案卷（卡）內作答

1. (5%) Find a base for the null space of the matrix $A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$.

2. (6%) Let S be the set of 2×2 singular matrices. Prove that S is not closed under addition, but is closed under scalar multiplication.

3. (7%) Consider that an invertiable matrix

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

maps the line $y = 2x + 1$ into another line $y\prime$. Find the equation of this line $y\prime$.

4. (7%) Consider the sequence of functions w_k, defined by $w_k(t) = \cos(kt)$ in the space $C[0, \pi]$ (consisting of all continuous functions on the interval $[0, \pi]$). Show that it is an orthogonal sequence.

5. (10%) Consider the vector $v = (3, 2, 6)$ in R^3. Let W be the subspace of R^3 consisting of all vectors of the form (a, b, b). Show that v can be expressed as the sum of a vector that lies in W and a vector that is orthogonal to W.

6. (15%) Consider that a Markov chain with an $n \times n$ transition matrix A converges to a steady-state vector x.

   (a) (10%) Prove that $\lambda_1 = 1$ is an eigenvalue of A, and x is an eigenvector belonging to λ_1.

   (b) (5%) Assume that A is diagonalizable and $\lambda_1 = 1$ is a dominant eigenvalue of A. Prove that the Markov chain with transition A converges to a steady-state vector.

參考用

注意：背面有試題

7. **(15%)** The number of hits at the CENews web site in any time interval is a Poisson random variable. The site has on average 3 hits per second. Let X be the number of hits in 0.5 seconds. Let Y be the number of hits in 2 seconds.
   (1) (5%) Find $P[Y \geq 2]$.
   (2) (5%) Find the expected value of X.
   (3) (5%) Find the variance of X.

8. **(15%)** Random variables X and Y have the joint probability density function (PDF)

$$f_{X,Y}(x,y)=\begin{cases}1/15 & 0\leq x\leq 5, 0\leq y\leq 3,\\ 0 & \text{otherwise.}\end{cases}$$

   (1) (5%) For $0\leq y\leq 3$, find the conditional PDF $f_{X|Y}(x\,|\,y)$.
   (2) (5%) Find the conditional PDF of X and Y given the event $B=\{X>Y\}$.
   (3) (5%) Let $W=XY$. Find the conditional expected value of W given the event $B=\{X>Y\}$.

9. **(15%)** Let $\mathbf{X}$ be the random vector $[X_1 \quad X_2 \quad X_3 \quad X_4]^T$, where X_1, X_2, X_3, and X_4 are independent and identically distributed (iid), and each continuous random variable X_i is uniformly distributed over the interval (0,1).
   (1) (5%) Find the expected vector of $\mathbf{X}$.
   (2) (5%) Find the correlation matrix $\mathbf{R}_\mathbf{X}$.
   (3) (5%) Find the covariance matrix $\mathbf{C}_\mathbf{X}$.

10. **(5%)** Random variables X and Y have the joint probability mass function (PMF):
$P_{X,Y}(1,1)=0.3$, $P_{X,Y}(1,2)=0.2$, $P_{X,Y}(1,3)=0.1$, $P_{X,Y}(2,1)=0.2$, $P_{X,Y}(2,2)=0.1$, $P_{X,Y}(2,3)=0.1$. What is the moment generating function of Y?

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