1. (20%, 2% each) Friction factor for tube flow

The friction factor \( f \)-Reynolds number \( \text{Re} \) relationships for a Newtonian fluid flowing through smooth conduits or commercial rough conduits are summarized in Figure A. The friction factor and Reynolds number are defined as follows as the conduits are circular:

\[
f = \frac{1}{4} \left( \frac{D}{L} \right) \left( \frac{P_0 - P_L}{\frac{1}{2} \rho(u)^2} \right) \quad (1-1)
\]

\[
\text{Re} = \frac{D(u) \rho}{\mu} \quad (1-2)
\]

And \( D, (u), \rho, \mu, P_0-P_L, \) and \( L \) are tube diameter, mean velocity, liquid density, liquid viscosity, pressure drop, and tube length, respectively. The symbol \( k \) in Figure A is the height (roughness) of the protuberances. Answer the following equations briefly:

(a) What is the physical meaning of Reynolds number?
(b) What is the physical meaning of friction factor?
(c) If the tubes are noncircular, what the Reynolds number (equation 1-2) and friction factor (equation 1-1) become?
(d) Why does the friction factor monotonously decrease with increases in Reynolds number? Does it indicate a reduction of shear stress when fluid velocity is increased?
(e) Why in laminar flow region friction factor is inversely proportional to Reynolds number?
(f) Why in turbulent flow region friction factor is independent of Reynolds number?
(g) Why in laminar flow region friction factor is independent of the roughness?
(h) Why in turbulent flow region friction factor increases with increases in wall surface roughness of conduits?
(i) Explain why in the turbulent flow region, the friction factor is affected by dimensionless group \( k/D \), but not by \( k \)?
(j) What is the physical meaning of "hydraulic smooth" limit in Figure A?
2. (10%, 2% each) **Stream Functions and Velocity Potential**

Comment and discuss briefly the following statements (and/or equations). Also please indicate whether each of these statements (and/or equations) is correct or incorrect.

(a) Stream functions ($\psi$) can be used to resolve the 2-dimensional flow problems for both compressible and incompressible fluids.

(b) Velocity potential ($\Phi$) is only used to solve 2-dimensional irrotational flow problems, and invalid for solving 3-dimensional irrotational flow.
(c) If the velocity potential is $\Phi = (5/3)x^3 - 5xy^2$, then its corresponding stream function is $\psi = 5x^2y$.

(d) The function, $F(x, y) = u_0L[x^2/L^3 - (3xy^2)/L^3]$, can be used as velocity potential to solve irrotational flow problems.

(e) The stream function can be used in solving both viscous and ideal (or inviscid) flow problems.

3. (10%)
(a) (4%) Write down the definitions of Nusselt number and Biot number. What is the difference between Nusselt number and Biot number?

(b) (6%) What dimensionless number(s) influence Nusselt number? Please also write down the definition of the dimensionless number(s).

4. (20%) Heat Conduction in an Annulus
Heat is flowing through an annular wall of inside radius $r_0$ and outside radius $r_1$. The temperature at $r = r_0$ is $T_0$ and the temperature at $r = r_1$ is $T_1$. The thermal conductivity varies linearly with temperature from $k_0$ at $T_0$ to $k_1$ at $T_1$. Develop an expression for the heat flow through the wall.

5. (40%) When a single-component liquid droplet evaporates into air as in the operation of spray-drying, or when a solid drug granule, modeled as a single-component sphere, dissolves in a liquid as in drug dissolution, we can construct a simple model of the diffusive transport that occurs between the object and the surrounding fluid.
You may assume: (1) The sphere contains a pure solid component A; therefore, you need to consider the mass transport process only in the surrounding fluid, (2) the fluid is unbounded in extent and quiescent. It contains only the diffusing species A, and a non-transferring species B, (3) the motion of arising from diffusion can be neglected. This requires that either the mixture in the fluid be dilute in species A, consisting primarily of the non-transferring species B, or that the rate of mass transport be small, (4) the problem is spherically symmetrical, there are no gradients either in the polar coordinates or the azimuthal angular coordinates, (5) after an initial transient, a steady state is assumed to prevail. The change in size of sphere due to mass transfer occurs on a time scale that is very large compared with the time scale for the diffusion process for a given radius of the sphere to reach steady state, and (6) there are no chemical reactions.

Please calculate:

(a) (5%) the flux of species A, $N_A(r)$,
(b) (5%) the molar rate of mass transfer at the surface of the sphere, $W_A$,
(c) (5%) the rate of change of the radius of the sphere with time, $a(t)$, assuming the mole fraction of species A in the fluid at the interface is $X_{A,f}$, the radius at time zero is $a_0$, the molecular weight of A be $M_A$, and the density of the sphere be $\rho$.

What is the advantage of making $a_0$ very small by nanotechnology?

(d) (5%) the flux of species A, $N_A(a_0)$, at the surface of a sphere of radius $a_0$,
(e) (5%) the time estimated for the diffusion process around a sphere of radius $a_0$,
(f) (5%) the time estimated for the sphere to completely dissolve in the fluid,
(g) (5%) what is the ratio of the two time scales calculated in Parts (e) and (f)?
(h) (5%) what should the ratio (i.e. the dimensionless group) in Part (g) become to make Assumption (5) in the problem statement valid?