國立中央大學八十六學年度碩士班研究生入學試題卷

所別：化學工程研究所　不分組　科目：　工程數學　　共 1 頁 第 1 頁

1. (a) Prove that the differential equation $(1+x^2)\frac{dy}{dx}+2xy=0$ is separable, and solve it accordingly. (5%)
   (b) Solve a typical Bessel's equation $x^2y'' + xy' + (x^2 - \frac{1}{9})y = 0$ (in terms of J_v and J_{-v}) (5%)
   (c) Find a second-order homogeneous linear differential equation for which $e^x\cos x$ and $e^x\sin x$ are solutions. Find the Wronskian of the functions and use it to verify the linear dependence or independence of the functions.(5%)
   (d) Find the general solution of $x\frac{dy}{dx}=y^2$, $y(1)=1$. (5%)

2. The two linear independent function $y_1(x)$ and $y_2(x)$ are solution of the homogeneous part of the following equation, and $y_p(x)$ is the particular solution of the nonhomogeneous equation. Prove that $c_1y_1(x)+c_2y_2(x)+y_p(x)$ is the general solution of the equation.

$$y'' + a(x)y' + b(x)y = f(x) \qquad (15\%)$$

3. (a) Does the inverse for matrix **A** exist? If yes, calculate the inverse matrix of **A**. If not, explain why. (7%)
   (b) Find the eigenvalues and eigenvectors for matrix **B**. (8%)

$$\mathbf{A}=\begin{bmatrix}-1 & 1 & 2\\ 3 & -1 & 1\\ -1 & 3 & 4\end{bmatrix} \qquad \mathbf{B}=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

4. (a) Find the directional derivative of f at point P in the direction of $\mathbf{a}$, where
   $f=x^2+y^2+z^2$,　　$P=(1,2,3)$,　　$\mathbf{a}=\mathbf{i}+\mathbf{j}$　　(7%)
   (b) (8 points) Compute $\int_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$, where
   $\mathbf{F}=y^2\mathbf{i}-x^2\mathbf{j}$,　　C: $y=2x^2$, $0\le x\le 1$　　(8%)

5. Write the Fourier series corresponding to the function

$$F(x)=\begin{cases}0 & -2<x<-1\\ k & -1<x<1\\ 0 & 1<x<2\end{cases} \qquad \text{period}=4 \qquad (10\%)$$

How should F(x) be defined at $x=-2$, $x=-1$, $x=0$, $x=1$ and $x=2$ in order that Fourier series will converge to F(x) for $-2\le x\le 2$? (5%)

6. A thin bar of length L in which heat transfer between the bar and its surrounding is assumed to obey a linear transfer law. The heat equation is then

$$\partial u/\partial t = a^2\,(\partial u^2/\partial x^2) - hu,$$

The ends of the bar are insulated and the initial temperature is f(x). Find the temperature distriubution u(x,t). (20%)