國立中央大學九十二學年度碩士班考試入學招生試題卷

系所別： 化學工程與材料工程學系 科目： 工程數學

(1) **(20 points)** If heat is generated throughout a laterally insulated bar of length π at a constant rate **H** per unit length and ends at x = 0 and x = π kept at temperature zero. Find the temperature u(x, t) in this bar with initial temperature f(x).

(2) **(15 points)** Assuming that the Fourier series corresponding to f(x) on the interval $\pi < x < -\pi$, $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$,converges in the mean to f(x). Prove

(a) Parseval's indenity

$$2a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)^2\,dx$$

(b) $\lim_{n\to\infty} \mathbf{a}_n = \lim_{n\to\infty} \mathbf{b}_n = 0$

(3) **(14 points)** 假設 A 、B 、C 是任意 3×3 square matrices, k 是 non-zero scalar. 以下諸式中，有那些恆成立？ 有那些不是恆成立？ 請對不是恆成立的式子舉實例說明。

(a) A B = B A

(b) If A B = 0, then A = 0 or B = 0

(c) (k A) B = k (A B) = A (k B)

(d) A (B C) = (A B) C

(e) (A + B) C = A C + B C

(f) C (A + B) = C A + C B

(g) $(A\,B)^{-1} = A^{-1}\,B^{-1}$

(4) **(6 points)** Find the eigenvalues and eigenvectors for the following matrix

$$\begin{bmatrix} 6 & 10 & 6 \\ 0 & 8 & 12 \\ 0 & 0 & 2 \end{bmatrix}$$

(5) **(10 points)** Find the length of the following curve r(t) = cos(t) $\underline{i}$ + sin(t) $\underline{j}$ + t $\underline{k}$ from (1, 0, 0) to (1, 0, 2π).

注意：背面有試題

(6) Quantum mechanics description of matters can be understood in term of Schrodinger equation, which is a partial differential equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi - V\Psi = E\Psi$$

(6.1)(7 points) In the case of hydrogen atom, we are dealing with a spherical coordinate system, and the final form of this equation is

$$-\left[\frac{\hbar^2}{2m}\right]\nabla^2\Psi - \left[\frac{Ze^2}{4\pi\varepsilon_0}\right]\frac{\Psi}{r} = [E]\,\Psi$$

where all terms in the square brackets are constants. For such a spherical coordinate system, the analysitac solution would involve (multiple choice):

(a) Legendre polynomial (b) Bessel function (c) Fourier series (d) Laguerre polynomial

(6.2)(8 points) In the case of a particle in a cubic box, the Schrodinger equation will be written in Cartesian coordinate and becomes

$$-\left[\frac{\hbar^2}{2m}\right]\nabla^2\Psi = E\Psi$$

with boundary conditions of $\Psi = 0$ @ $x = \pm L$, $y = \pm L$ and $z = \pm L$. The analytical solution of this equation will linvolve (multiple choice):

(a) Legendre polynomial (b) Bessel function (c) Fourier series (d) Laguerre polynomial

(7)(10 points) Try all the possible methods you know on solving first order differential equation on the equation given below.

$$\frac{dy}{dx} = 2\frac{\sin(y^2)}{xy\cos(y^2)} + x, \text{ IC } y(2) = \sqrt{\pi/2}$$

Give reason on why that particular method works or why it does not apply. Each method and explanation will be credited.

(8)(10 points) Solve the following differential equation and show the details of your derivations.

$$y'' - 4y' + 4y = \frac{e^{2x}}{x},\ y(1) = 0,\ y'(1) = 1$$