

所別：土木工程學系碩士班 庚組 科目：工程數學

Part A

- 一、輔以平面銳角三角形圖示，試明列餘弦定律邊角之關係和其推導過程(16%)。
- 二、存在某向量 \bar{a} ，內含 $(a_1, \dots, a_i, \dots, a_n)$ 個實數元素；試
 - (一)定義該向量之長度 $|\bar{a}|$ ，並
 - (二)詳列偏導數 $\partial|\bar{a}|/\partial a_i$ 之公式(17%)。
- 三、已知線性誤差方程組與量測誤差協方差(Covariance)矩陣： $\mathbf{v} + \mathbf{Ax} = \mathbf{l}$ 與 Σ ，於此矩陣皆無秩虧之虞。經間接觀測最小二乘平差能得向量估計解為 $\hat{\mathbf{x}}$ 及 $\hat{\mathbf{v}}$ ；試證明雙線性形等於零： $\hat{\mathbf{v}}^T (\Sigma^{-1} \mathbf{A}) \hat{\mathbf{x}} = 0$ (17%)。

注意：背面有試題

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Part B

4. A 3D surface can be represented in a parametric form as

$$x = f(u, v), \quad y = g(u, v), \quad z = h(u, v) \quad (4-1)$$

In a spherical coordinate system, this becomes

$$x = \rho \sin \varphi \cos \theta, \quad y = \rho \sin \varphi \sin \theta, \quad z = \rho \cos \varphi \quad (4-2)$$

where ρ is the radius.

- What is the effective range of φ and θ ? (4%)
- Find the Jacobian matrix of this spherical surface. (Hint: try to linearize Eq. (4-1) or (4-2) by tracking differentials.) (7%)
- Also find the normal vector at any given point on this surface. (6%)

5. The Fourier transform, $F(u)$, of a single variable continuous function, $f(x)$ is defined as

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx \quad (5-1)$$

where $j = \sqrt{-1}$; and the inverse Fourier transform is

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du \quad (5-2)$$

A Gaussian lowpass filter in the frequency domain has the transfer function

$$H(u, v) = A e^{-(u^2 + v^2)/2\delta}$$

Show that the corresponding filter in the spatial domain has the form

$$h(x, y) = A 2\pi \delta^2 e^{-2\pi^2 \delta^2 (x^2 + y^2)}$$

(Hint: Treat the variables as continuous and $(u^2 + v^2)$ can be replaced by a distance square.) (17%)

6. What does the equation $AX = B$ mean to you? How will you solve the equation? (16%)