

國立中央大學101學年度碩士班考試入學試題卷

所別：機械工程學系碩士班 丁組(系統)(一般生)

科目：工程數學(含程式設計) 共 2 頁 第 1 頁

機械工程學系光機電工程碩士班 甲組(機電系統控制)(一般生) 科目：工程數學及程式設計

本科考試可使用計算器，廠牌、功能不拘

*請在試卷答案卷(卡)內作答

Vector analysis and linear algebra

- Given a scalar function $f(x, y) = x^2 + y$,
 - Find the direction along which the directional derivative of f at $(2, 1)$ is a maximum. (5%)
 - Find the directional derivatives of f at $(2, 1)$ in the direction parallel to the tangent vector of the curve $x^2 + 2y^2 = 9$ at $(1, 2)$. (5%)
- Given a vector field $\mathbf{F}(x, y) = x^2\hat{i} + y\hat{j}$, find the direction along which $\left| \frac{d\mathbf{F}(2,1)}{ds} \right|$ is a maximum, where s is the displacement. (8%)
- Consider the linear system of equations $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & m & 2 \\ 2 & n & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}.$$

Find the relation of m and n for which the problem has solution(s). (7%)

Partial differential equations and complex analysis

- Find the deflection of the membrane of sides \mathbf{a} and \mathbf{b} with $c^2 = 1$ ($c^2 = \frac{T}{\rho}$, where T is tension and ρ is the mass per unit length) from the two dimensional wave equation for the initial deflection $f(x, y) = xy(a-x)(b-y)$ and initial velocity 0. Deflections are zero on the boundary. (15%)
- Flow with circulation around a cylinder can be represented by the complex potential : $F(z) = z + \frac{1}{z} - \frac{iK}{\pi} \ln z$. Calculate the stagnation point (points at which the velocity is zero) for $K = 2\pi$ and 4π , respectively for which the cylinder wall $|z|=1$ is a streamline. (10%)

Ordinary differential equations

- Consider the differential equation

$$x^2 y'' + (3x - 1)y' + y = 0.$$

- Show that $x = 0$ is an irregular singular point. (5%)
- Show that

$$y = \sum_{n=0}^{\infty} n! x^n$$

is a series solution of the given differential equation. (5%)

注意：背面有試題

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7. A rabbit starts at the origin and runs up the y-axis with speed a . At the same time a dog, running with speed b , starting at the point $(c,0)$ and pursues the rabbit. Show that the path of the dog can be mathematically expressed by the differential equation,

$$xy'' = \frac{a}{b} \sqrt{1+(y')^2}. \quad (10\%)$$

8. Let a sequence of functions $\phi_1(x), \phi_2(x), \dots, \phi_n(x), \dots$ be orthonormal functions on an interval $a \leq x \leq b$. Verify that

$$a_k = \int_a^b f(x)\phi_k(x)dx, \quad k = 1, 2, \dots, n, \dots$$

where $f(x)$ is an analytical function and a_k is the coefficient of expansions of the form representing $f(x)$ such that

$$f(x) = \sum_{k=1}^{\infty} a_k \phi_k(x) \quad (5\%)$$

程式語言

9. 數學中常用的 π 可用下列無窮數列來計算：

$$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \dots$$

吾人可藉由計算數列的一項、二項、三項和...，來求得 π 的近似值。試寫一程式，以迴圈方式計算至 n 項和之 π 的近似值，並將結果儲存於變數 result 中。程式碼限定以 C、C++、Visual Basic 或 Fortran 撰寫，除迴圈變數以整數宣告外，其餘所有變數均以實數宣告，並註明使用的程式語言。 (10%)

10. 在程式語言中，常用一、二維陣列來分別表示方向向量及轉換矩陣。假設有兩個方向向量 P、R 與一個轉換矩陣 T，其維數依序是 1×4 、 1×4 、 4×4 。若 $R = P \times T$ ，如此可藉由轉換矩陣 T 將方向向量 P 在空間中平移或旋轉到方向向量 R。若 P、T 為已知，試寫一程式計算轉換後之方向向量 R，程式中需使用迴圈計算，程式碼限定以 C、C++、Visual Basic 或 Fortran 撰寫，除迴圈變數以整數宣告外，其餘所有變數均以實數宣告，其數值無需考慮，並註明使用的程式語言。 (15%)

注意：背面有試題