國立中央大學107學年度碩士班考試入學試題

所別： 機械工程學系 碩士班 製造與材料組(一般生)　　共2頁　第1頁
機械工程學系光機電工程 碩士班 光機組(一般生)
能源工程研究所 碩士班 不分組(一般生)

科目： 工程數學

本科考試可使用計算器，廠牌、功能不拘　　*請在答案卷(卡)內作答

1. Solutions for ordinary differential equations (ODEs) (25%)

(a) Find the solution for the ODE $y'=(x+y-2)^2, y(0)=2$. (Hint: set $v=(x+y-2)$) **(7%)**

(b) Find the solution for the 2nd-order ODE $x^2y''-xy'+y=0, y(1)=1.5, y'(1)=0.25$. **(8%)**

(c) Find the solution of the initial value problem
$y''+3y'+2y=10[\sin t+\delta(t-1)],\ y(0)=1,\ y'(0)=-1$. $\delta(t-1)$ is Dirac delta function. **(10%)**

2. Vector analysis and Linear algebra (25%)

(a) Please find the parametric equations of streamline through $(-1, 6, 2)$ for the vector $\mathbf{F}(x, y, z)=x^2\mathbf{i}+2y\mathbf{j}-1\mathbf{k}$ (x and y are not zero) using the equations $\frac{dx}{x^2}=\frac{dy}{2y}=\frac{dz}{-1}$ **(10%)**

(b) Please solve the following nonhomogeneous systems of ODEs by evaluating (i) matrix form, $J'=AJ+g$ **(2%)**; (ii) eigenvalues and eigenvectors of matrix A **(5%)**; (iii) the corresponding homogeneous and nonhomogeneous solutions **(8%)**

$$\begin{cases} I_1'=-4I_1+4I_2+12 \\ I_2'=-1.6I_1+1.2I_2+4.8 \end{cases}$$

3. Laplace transform / Fourier analysis (25%)

The Fourier transform pairs of two time signals $f(t)\leftrightarrow F(\omega)$ and $g(t)\leftrightarrow G(\omega)$ denote $F(\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt$ and $G(\omega)=\int_{-\infty}^{\infty}g(t)e^{-j\omega t}dt$, where t and ω are time and angular frequency, respectively. It's known that two properties of the Fourier transform are called (i) the time shift theorem, that is $f(t-t_0)\leftrightarrow F(\omega)e^{-j\omega t_0}$, and (ii) the convolution theorem, that is $f(t)*g(t)\leftrightarrow F(\omega)G(\omega)$, where $f(t)*g(t)=\int_{-\infty}^{\infty}f(\tau)g(t-\tau)d\tau$.

(a)Here *f(t)* is a time signal.

(i) **(6%)** Derive (or prove) the Fourier transform of $f(t-t_0)$ to be $F(\omega)e^{-j\omega t_0}$.

(ii) **(3%)** If a specific time signal was defined as $f(t)=\begin{cases} t, & 0\le t<1 \\ 2-t, & 1\le t<2 \end{cases}$. Sketch *f(t)* and *f(t-1)*.

(iii) **(3%)** Give the physical meaning of (or give interpretation to) *f(t)* and *f(t-1)* in both the time domain and the frequency domain ($F(\omega)e^{-j\omega t_0}$).

(b) Here *f(t)* and *g(t)* are both time signals.

(i) **(10%)** Derive (or prove) the Fourier transform of *f(t)*g(t)* to be *F(ω)G(ω)*.

(ii) **(3%)** If now *f(t)* is the input guided into a system with the impulse response function *g(t)*, please give the interpretation to the convolution theorem. (To make a sketch for explanation is helpful.)

注意:背面有試題

參考用

國立中央大學107學年度碩士班考試入學試題

所別： 機械工程學系 碩士班 製造與材料組(一般生)　　共2頁　第2頁
機械工程學系光機電工程 碩士班 光機組(一般生)
能源工程研究所 碩士班 不分組(一般生)

科目： 工程數學

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷(卡)內作答

4. Partial differential equations (PDEs) (25%)

The striking hammer problem could utilise the following mathematical notation to consider,

$$u_{tt} = c^2 u_{xx} + s(x,t),\ 0 < x < L,\ t > 0$$

$$u(0,t) = u(L,t) = 0$$

$$u(x,0) = u_t(x,0) = 0.$$

If the striking hammer is not perfectly rigid, then its effect must be included as a time-dependent forcing term of the form:

$$s(x,t) = \begin{cases} v \cdot cos\left(\dfrac{\pi(x-\xi)}{2d}\right) \cdot sin\left(\dfrac{\pi t}{\delta}\right), for\ |x-\xi| < d, 0 < t < \delta \\ 0 \qquad , otherwise. \end{cases}$$

Please find the general solution **(10%)** and motion of the string for $t > \delta$ **(15%)**.

注意:背面有試題

參考用