國立中央大學111學年度碩士班考試入學試題

所別： 機械工程學系 碩士班 製造與材料組(一般生)
機械工程學系光機電工程 碩士班 光機組(一般生)
能源工程研究所 碩士班 不分組(一般生)

科目： 工程數學

1. (10%) The Legendre polynomials, $P_m(x), m = 0, 1, 2, \cdots$ are given by $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{1}{2}(3x^2 - 1)$, $P_3(x) = \frac{1}{2}(5x^3 - 1)$, and so forth. It is known that $P_m(x)$ is orthogonal to $P_n(x)$ for $m \neq n$ on interval $-1 \leq x \leq 1$. Expand $f(x) = 3x^2 - 2x + 2$ by Legendre polynomials, i.e., $\sum_{n=0}^{N} a_n P_n(x)$.

(a) (i) (3%) Find a_0, a_1, a_2, and a_3 as $N \to \infty$.

(ii) (2%) Is the expansion complete?

(b) (i) (3%)Find a_0, a_1, a_2, and a_3 as $N = 3$.

(ii) (2%) Is the expansion complete?

2. (15%) A function $f(t)$ is expanded using Fourier series.

$$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n cos(\omega_n t) + b_n sin(\omega_n t)], \omega_n = \frac{2\pi n}{T}, T = 2\pi \qquad (1)$$

, where

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, dt,\ a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\, cos(\omega_n t)dt,$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\, sin(\omega_n t)dt; n = 1,2, \cdots$$

Suppose the coefficients are given by $a_0 = 1, a_n = 0$ for $n > 0$, and $b_n = 2^{-n}, n > 0$.

(a) (3%) What are the (fundamental) period and fundamental frequency, respectively?

(b) (3%) $f(t)$ is an even function, an odd function, or neither?

(c) (3%) Find the average value of $f(t)$ over $t = [0, 2\pi)$.

(d) (3%) Does $f\left(t + \frac{3T}{2}\right)$ equal to $f(t)$? Briefly give your reasons within 20 words.

(e) Eq. (1) can be expressed as $f(t) = \sum_{n=0}^{\infty}[c_n cos(n\omega_0 t + \phi_n)], 0 \leq \phi_n < 2\pi$.
Find c_0 and c_1. (3%)

3. (15%) Solutions for ordinary differential equations (ODEs)

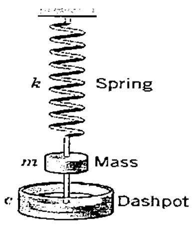
(a) (5%) Find the solution for the ODE: $y'' - k^2 y = 0\ (k \neq 0), y(0) = 1, y'(0) = 1$

(b) (5%) Find the solution for the ODE: $y'' + 4y' + 5y = e^{-t}cos(t), y(0) = 0, y'(0) = 1$

(c) (5%) Find a basis of solutions by the Frobenius method of the following ODE:

$$(x + 1)^2 y'' + (x + 1)y' - y = 0.$$

4. (10%) For the mass-spring system, as shown in the following figure, find its motion as a function of time, $y(t) =$? If the mass, m, is 0.25 kg, damping, c, is zero, spring constant, k, is 2.25 kg/sec^2, and driving force is $cos(t) - 2sin(t)$. Assuming zero initial displacement and velocity, $y(0) = 0, y'(0) = 0$. For what frequency of the driving force would you get resonance?



注意：背面有試題

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所別： 機械工程學系 碩士班 製造與材料組(一般生)　　共2頁　第2頁
機械工程學系光機電工程 碩士班 光機組(一般生)
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5. (5 %) Suppose that in a weight-watching program, Sam, weighing 180 lb, burns 350 cal/hr in walking (3 mph), 500 cal/hr in bicycling (13 mph), and 950 cal/hr in jogging (5.5 mph). He plans to exercise 4 days a week following to Mon (1.0, 0, 0.5), Wed (1.0, 1.0, 0.5), Fri (1.5, 0, 0.5), and Sat (2.0, 1.5, 1.0), where 3 numbers in each bracket give the time (hrs) taken to walk, bicycle, and jog, respectively, on that day. Please give the **(4 by 1) column vector** that shows the calories burned on Mon, Wed, Fri, and Sat in a week.

6. (20%)

(a) (5%) Find the inverse of the matrix, $\begin{bmatrix} 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} \\ \frac{1}{8} & 0 & 0 \end{bmatrix}$, by **Gauss-Jordan elimination.**

(b) (8%) Please explain or address why Gauss-Jordan elimination can be used to calculate the inverse of a matrix.

(c) Let $\boldsymbol{A} = [a_{jk}]$ be a nonzero square matrix of dimension $n \times n$. The problem of finding nonzero $\vec{x}$'s and λ's that satisfy the vector equation

$$\boldsymbol{A}\vec{x} = \lambda\vec{x} \qquad (2)$$

is called an eigenvalue problem.

(i) (4%) Please interpret the meaning of Eq. (2).

(ii) (3%) What do you find from the matrix multiplication $\begin{bmatrix} 6 & 3 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \end{bmatrix}$?

7. (10%) A closed curve C on the x-y plane is made from the following three straight line segments:
S1: between (x,y)=(0,0) and (2,0)
S2: between (2,0) and (2,1)
S3: between (0,0) and (2,1)

Given $\phi = 3xy^2$, determine the line integral $\oint_C \nabla\phi \cdot d\vec{s}$ about the curve C, where ∇ is the gradient operator and $d\vec{s}$ is an infinitesimal displacement on C.

8. (15%) Let z satisfy the following two partial differential equations,

$$\frac{\partial^2 z}{\partial x \partial y} = 0 \text{ and } a\frac{\partial^2 z}{\partial r^2} = b\frac{\partial^2 z}{\partial s^2}.$$

Given $x + y = \sqrt{c}s$ and $x - y = dr$, determine a form of z together with the associated relation between the four constants a, b, c and d.

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