國立中央大學八十七學年度碩士班研究生入學試題卷

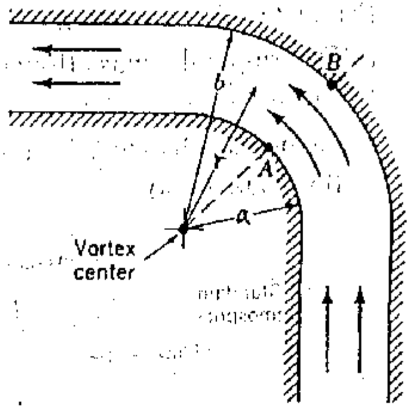
所別： 機械工程研究所 丙組 科目： 流體力學 共 2 頁 第 1 頁

1. (30%) (a) Explain the difference between a solid and a liquid. (3%)
 (b) What is the definition of the Newtonian fluid. (3%)
 (c) When is the flow termed irrotational? When may an irrotational flow become rotational? (6%)
 (d) Show that lines of constant velocity potential are orthogonal to lines of constant stream function. (6%)
 (e) When is the Bernoulli equation valid? Use the Bernoulli equation to explain the concepts of the energy line and the hydraulic grade line. (8%)
 (f) Describe the difference between the Eulerian and Lagrangian methods. (4%)

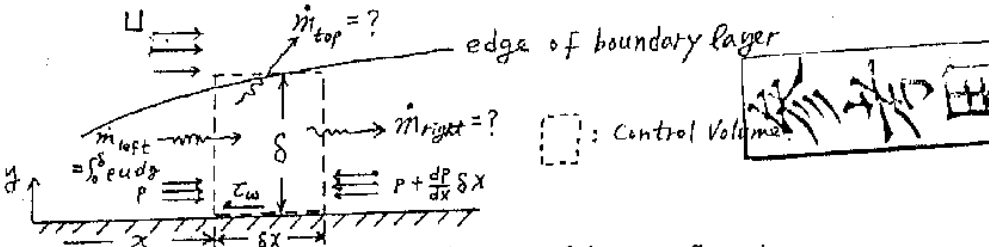
2. (15%) The velocity distribution in a horizontal two-dimensional 1/4 circular bend through which the flow can be approximated with a free vortex.
 (a) Write out the expression of velocity at the bend section. (3%)
 (b) Write out the expression of pressure along line A-B. (4%)
 (c) Proof that the volume flow rate per unit width (q) through the bend can be expressed as

$$q = C\sqrt{\frac{\Delta p}{\rho}}\text{, where } \Delta p = p_A - p_B\text{. (4\%)}$$

 (d) Determine the value of C in terms of a and b. (4%)



3. (26%) **Boundary Layer**
 A. Define the following key boundary layer parameters: (a) The boundary layer thickness (δ); (b) the displacement thickness (δ^*); (c) the momentum thickness (Θ); (d) the wall shear stress (τ_ω); (e) the skin friction coefficient. (each 2 %)
 B. The figure below shows a control volume that spans the entire boundary layer, extending from the wall to a height (δ). The length of the control volume is δx. The control volume has unit depth perpendicular to the paper.



注意：背面有試題

 (a) Write the continuity equation for the control volume in terms of the mass flux $\dot{m}_{left}$, $\dot{m}_{right}$, and $\dot{m}_{top}$. What is $\dot{m}_{top}$? (2 %)
 (b) What is the x-momentum flux at the right and top faces ($\dot{M}_{right}$ and $\dot{M}_{top}$)? (2 %)
 (c) What is the x-direction Momentum equation in terms of $\dot{M}_{x,left}$, $\dot{M}_{x,right}$, $\dot{M}_{x,top}$, $F_{x,pressure}$, and $F_{x,stress}$? (2 %)
 (d) Using the above relations, prove the so-called von Karman's momentum integral equation: $\frac{d}{dx}\int_0^\delta u(U-u)dy + \frac{dU}{dx}\int_0^\delta (U-u)dy = \frac{\tau_\omega}{\rho}$. (5 %)
 (e) Derive the von Karman's momentum integral equation in terms of δ^*, Θ, U, and τ_ω. (5 %)

4. (9%) **Dimensional Analysis**

A. What is the dynamic similarity? (3 %)

B. A vortex ring is a toroidal loop of vorticity which is formed by an impulse at $t = 0$. The impulse of the ring $I = \rho U \delta^3 = \rho \omega \delta^4$, where $\omega = U/\delta$, is thereafter constant. How do U and δ depend on z, if the ring is self-similar? (Note that z is the propagating direction of the vortex ring.) (6 %)

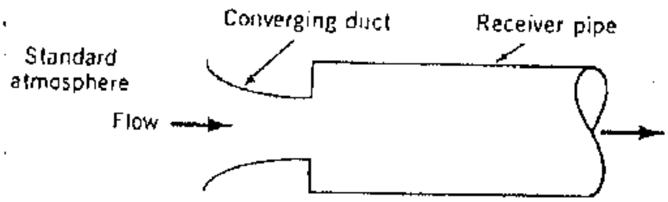
5. (20%) A converging nozzle passes air steadily from standard atmopheric conditions (p_0=1 atm, T_0=15°C, ρ_0=1.23 kg/m^3) to a receiver pipe. The area of throat of the converging duct is 1×10^{-4} m^2. Assume the flow in the converging nozzle is isentropic. The receiver pressure is 40 kPa. (R=286.9 J/kg.K).

(a)Determine the pressure at the exit of nozzle. (5%)

(b) Determine the Mach no. at the exit of nozzle. (3%)

(c)Determine the mass flow rate through the nozzle. (8%)

(d) If the pressure of receiver pipe decreases to 20 kPa, will the mass flow rate through the nozzle changes? You only need to briefly explain and need not to calculate the mass flow rate. (4%)



(a)