(1) If F(s) is the Laplace transform of f(t) and $F(s)=\dfrac{s+1}{s^2+s-6}$. Find f(t). (25%)

(2) The gradient of a scalar function f is defined as

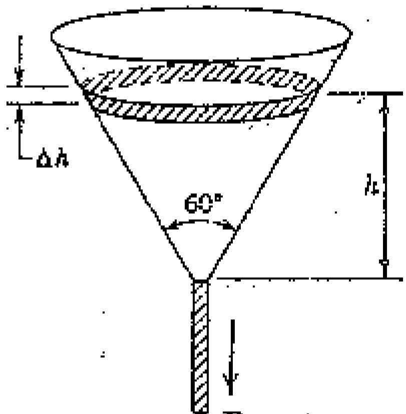
$$\nabla f=\frac{\partial f}{\partial x}\mathbf{i}+\frac{\partial f}{\partial y}\mathbf{j}+\frac{\partial f}{\partial z}\mathbf{k}$$

The divergence of a differentiable vector function **V** is defined as

$$div\,\mathbf{V}=\frac{\partial v_1}{\partial x}+\frac{\partial v_2}{\partial y}+\frac{\partial v_3}{\partial z}$$

where v_1, v_2 and v_3 are components of **V**. Express $div(f\mathbf{V})$ as a function of f, **V**, $div\mathbf{V}$ and ∇ f. (25%)

(3) A funnel, as shown in the figure, whose angle at the outlet is 60° and whose outlet has a cross-sectional area of 0.5 cm^2, contains water. At time t=0 the outlet is opened and the water flows out. Determine the time when the funnel will be empty, assuming that the initial height of water is h(0)=10 cm. The velocity with which a liquid issues from an orifice is $v=0.6\,(2gh)^{1/2}$. (25%)



Funnel

(4) A semi-infinite solid, $0 \le x < \infty$, is initially at zero temperature. For times t>0 the boundary surface at x=0 is kept at temperature f(t)=cost. (25%)

(a) Write down the governing equation and boundary/initial conditions for the problem.

(b) Obtain an expression for the temperature distribution T(x,t) in the solid for times t>0