所別：光機電工程研究所碩士班 不分組 科目：工程數學及程式設計

Ordinary Differential Equation (25%)

1. (a) Please use Laplace Transform to solve the ordinary differential equation of $y'' + 25y = 5\delta(t-\pi)$, with initial conditions of $y(0) = 3$ and $y'(0) = 0$. Note that δ is the Dirac delta function. (12%)

   (b) Calculate the values of $y(\pi/2)$ and $y(2\pi)$. (3%)

2. Solve $y' + y = -2x/y$ with initial condition of $y(0) = 2$. (10%)

Linear Algebra & Vector Calculus (25%)

3. For the linear system $\mathbf{Ax} = \mathbf{b}$, where the matrix $\mathbf{A} = [a_{ij}]_{3\times4}$ is given by

$$\mathbf{A} = \begin{bmatrix} -1 & 5 & -1 & -3 \\ 4 & -1 & 2 & 6 \\ 3 & 4 & 1 & 3 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

   (a) Find all the possible vectors $\mathbf{b}$ for which the linear system has non-trival solution. (5%)

   (b) Determine the solution $\mathbf{x}$. (5%)

4. Use Green'theorem to evaluate $\oint_C \vec{F} \cdot d\vec{R}$, where $\vec{F} = xy\vec{i} + xy^2\vec{j}$ and C: the triangle with vertices $(0,0)$, $(3,0)$, $(0,5)$. Note that the curve C is oriented counterclockwise. (8%)

5. Let $\{v_1, v_2\}$ span the vector space of inner product in R^2. Please answer the following questions.

   (a) Is it true that v_1 and v_2 must be mutually orthorgonal? Explain why or why not. (3%)

   (b) Give two examples showing that $\{v_1, v_2\}$ is a orthornormal base in R^2. (4%)

注意：背面有試題

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Fourier Analysis, Partial Differential Equation and Complex Analysis (25%)

6. The function

$$f(x)=\begin{cases}-1, & -\pi<x<0\\ 1, & 0\le x<\pi\end{cases}$$

(a) Expand $f(x)$ in a Fourier series. (5%)

(b) Expand $f(x)$ in a complex Fourier series. (5%)

7. (a) Solve the partial differential equation (5%)

$$\frac{\partial u}{\partial x}+3\frac{\partial u}{\partial y}=0$$

(b) Solve the boundary-value problem (10%)

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t},\qquad 0<x<\pi,\qquad t>0$$
$$u(0,t)=0,\qquad u(\pi,t)=0,\qquad t>0$$
$$u(x,0)=\sin x,\qquad 0<x<\pi.$$

程式語言 **(25 %)**

8. 以下三行 C 語言程式碼，請解釋變數 A 的意義 (5 %)

```
double *A;
int num=100;
A=(double *)malloc(num*sizeof(double));
```

9. 擬使用"struct"自訂一資料結構，內有三個變數， P、Q 與 R，其中 P 爲單精度實數，Q 爲雙精度實數，R 爲整數，請以 C 語言寫出此一資料結構。(8 %)

10. 變數 i 爲整數，x 爲實數，i 由 0 到 10，x 則爲以下的函數：(12%)

(1) 若 i<5，則 x=i*i

(2) 若 i>=5，則 x=i*i*i

請使用 C 語言，以迴圈方式寫出上述的程式碼。