國立中央大學 109 學年度碩士班考試入學試題

所別： 機械工程學系 碩士班 製造與材料組(一般生)　　共 1 頁 第 1 頁
機械工程學系光機電工程 碩士班 光機組(一般生)
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科目： 工程數學

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷(卡)內作答
計算題需計算過程，無計算過程者不予計分

I. Solutions for ordinary differential equations (ODEs)

A. Find the solution for the ODE $e^{3\theta}(dr + 3rd\theta) = 0.$ (4 %)

B. Find the solution for the ODE $y'' + 16y = 4sint,\ y(0) = 0, y'(0) = 1.$ (7 %)

C. Find a basis of solutions by the Frobenius method of the following ODE:
$x^2y'' + x(2x-1)y' + (x+1)y = 0.$ (8 %)

D. Find a general solution for the ODE of $x^3y''' + 2x^2y'' - xy' + y = -x^2.$ (6 %)

II. **Definition**: The Laplace transform of a function $f(t)$ is written as $\mathcal{L}(f)$.
Suppose that $\mathcal{L}(f)$ exists. Does $\mathcal{L}(f^t)$ exists? Explain the reasons within 40 words. (5%)

III. **Definition**: Denote $\delta(t)$ as the Dirac's Delta function.
An ODE is written as $y'' + 4y' + 3y = \delta(t-2)$ with initial conditions $y(0) = y'(0) = 0.$
Solve the ODE by Laplace transform: (8%)

IV. **Definition**: The Fourier series expansion of a function $f(t)$ is given by

$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n cos(n\omega_0 t) + b_n sin(n\omega_0 t)], \omega_0 = \frac{2\pi}{T}.$

A function $f(t) = 2\cos^2(t), t \in R$ is expanded using Fourier series.

A. What is the (fundamental) period T of $f(t)$? (4%)

B. Find the values of a_0, a_1, a_2, b_1 and b_2 (8%)

V. Please solve the following linear systems

A. $\begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 130 \\ 60 \\ -240 \\ 50 \end{bmatrix}$ (5%); B. $\begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 45 \\ 40 \\ 55 \\ 50 \end{bmatrix}$ (5%).

VI. The stress at a given point can be evaluated by the following matrix，A= $\begin{bmatrix} 5 & 0 & 0 \\ 0 & -6 & -12 \\ 0 & -12 & 1 \end{bmatrix}$, please calculate

A. the principal values, (5%)

B. the principal directions (5%)

C. the angles between the three principal direction and the coordinate axes, for the given stress state (5%)

VII. Given $\vec{G} = y\vec{i} - z\vec{j} + yz\vec{k}$, please find the surface integral $A = \iint_S \vec{G}\cdot\vec{n}dA$, for

$S: x = \sqrt{y^2+z^2}, y^2 + z^2 \le 1.$ (15%)

VIII. Solve the boundary value problem $u_t = u_{xx} + \sin(3\pi x)$ with the boundary conditions: $u(0,t) = 0$, $u(1,t) = 0$ and initial condition: $u(x,0) = sin(\pi x).$ (10%)