

國立中央大學 108 學年度碩士班考試入學試題

所別： 環境工程研究所 碩士班 甲組(一般生)

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環境工程研究所 碩士班 乙組(一般生)

科目： 工程數學

本科考試禁用計算器

計算題需計算過程，無計算過程者不予計分

1. (20%) Show that

(i) $\mathcal{L}\left(\frac{df}{dt}\right) = sF(s) - f(0);$

(ii) $\mathcal{L}\left(\frac{d^2f}{dt^2}\right) = s^2F(s) - sf(0) - \frac{df(0)}{dt}.$

Where $F(s) = \mathcal{L}(f(t))$

2. (15%) Calculate the Fourier cosine series expansion of the function

$f(x) = \sin x$ in the interval $[0, \pi]$.

3. (25%) Let $f(x, y, z) = x^2 + y^3 + z^4 - 3$

(i) Calculate the directional derivative of f at $(1, 1, 1)$ in the direction $(1, 2, -3)$.

(ii) Calculate ∇f at the point $(1, 1, 1)$.

(iii) Calculate equation of a plane P tangent to f at $(1, 1, 1)$.

(iv) Calculate $\text{div}(\nabla f)$.

(v) Calculate $\text{curl}(\nabla f)$.

4. (15%) Solve the ODE

$$(y^2 + xy + 1)dx + (x^2 + xy + 1)dy = 0$$

5. (25%) Oxygen deficit (D) is a function of the competition between oxygen utilization and reaeration from the atmosphere:

$$\frac{dD}{dt} = k_d L - k_r D$$

Where $\frac{dD}{dt}$ is the change in oxygen deficit per unit of time

k_d is the deoxygenation rate constant

L is ultimate BOD of river water

k_r is reaeration rate constant

(i) Please integrate the equation and derive the *Streeter-Phelps* DO sag equation (using the initial condition: $D = D_a$ at $t = 0$).

(ii) The lowest point on the DO sag curve is called the critical point. Please find the time to the critical point (t_c).

參考用