所別：機械工程學系碩士班 甲組(固力與設計)(一般生)　科目：工程數學　共 2 頁 第 1 頁
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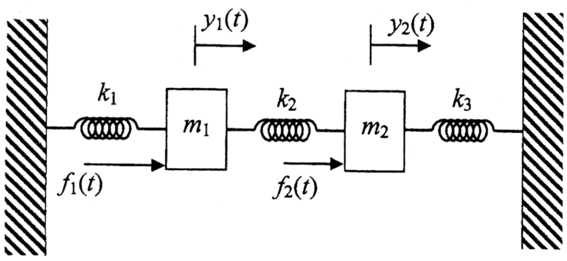
＊請在試卷答案卷（卡）內作答
＊本科考試可使用計算器，廠牌、功能不拘

Ordinary Differential Equation (25 %)

1. Solve the nonhomogeneous Euler-Cauchy equation (25%)

$$x^3y''' - x^2y'' - 7xy' + 16y = x^3 \ln x$$

Laplace/Fourier Transformation (25 %)



2. A mass-spring system as shown in above figure can be modeled in terms of the following equations of motion,

$$m_1\ddot{y}_1 + (k_1+k_2)y_1 - k_2y_2 = f_1(t)$$
$$m_2\ddot{y}_2 - k_2y_1 + (k_2+k_3)y_2 = f_2(t), \quad y_1(0) = \dot{y}_1(0) = y_2(0) = \dot{y}_2(0) = 0,$$

where m_1 and m_2 represent the mass, $k_1 \sim k_3$ are the spring constants, $f_1(t)$ and $f_2(t)$ are external forces, $y_1(t)$ and $y_2(t)$ represent the displacements of m_1 and m_2, respectively, and $(\ddot{\bullet})$ represents $\dfrac{d^2(\bullet)}{dt^2}$.

(a) Suppose $f_1(t) = \begin{cases} 2; & 0<t<3 \\ 0; & t>3 \end{cases}$, and $f_2(t) = \begin{cases} 0; & 0<t<2 \\ \delta(t-5); & 2<t<6 \\ 2te^{-3t}\cos(3\pi t); & t>6 \end{cases}$. Let $\mathcal{F}_1(s)$ represents the Laplace transform of $f_1(t)$. Find $\mathcal{F}_1(s)$. (5%)

(b) Express the function of time, $f_1(t)$, in terms of functions of frequency, $\alpha(\omega)$ and $\beta(\omega)$, by using the Fourier Integral representation of $f_1(t)$. (7%)

(c) Let m_1=1, m_2=1, k_1=4, k_2=5/2, and k_3=4.
Find the solution $y_1(t)$ and $y_2(t)$ for $0 < t < 4$. (13%)

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Linear Algebra (25 %)

3. Let C be the curve traversing the quarter-circle $x^2+y^2=1$ from $(1,0)$ to $(0,1)$ in the plane, then moving along the horizontal line from $(0,1)$ to $(2,1)$. Let $\vec{F}(x,y,z)=4x\vec{i}$. Compute $\int_C \vec{F}\cdot d\vec{R}$. (5%)

4. Consider the linear system equation $\mathbf{Ax}=\mathbf{b}$ where

$$\mathbf{A}=\begin{bmatrix}3 & 2 & 1\\ 2 & 1 & 1\\ 6 & 2 & m\\ 5 & 3 & 2\end{bmatrix} \quad \text{and} \quad \mathbf{b}=\begin{bmatrix}3\\ n\\ 6\\ 3+n\end{bmatrix}.$$

Let both m and n be real number, determine

(a) the rank of $\mathbf{A}$; (3%)

(b) the values of m and n, if the system equation has infinitely many solutions; (4%)

(c) the values of m and n, if the system equation has precisely one solution; and (4%)

(d) the values of m and n, if the system equation has no solution. (4%)

5. Please directly apply the theorem of Gauss, $\iiint_T \mathrm{div}\vec{F}dv=\iint_S \vec{F}\cdot\vec{n}dA$, to evaluate the integral $\iint_S (7x\vec{i}-z\vec{k})\cdot\vec{n}dA$ over the sphere $S: x^2+y^2+z^2=9$. (5%)

Partial Differential Equation+ Complex Analysis (25 %)

6. A certain function $f(z)$ is represented by the expansion (5%)

$$\frac{1}{z^3}+\frac{1}{z^4}+\frac{1}{z^5}+\cdots$$

in $1<|z|<\infty$. Determine the close form of the function, and evaluate $f(3i)$.

7. (a) Solve the partial differential equation (20%)

$$\frac{\partial u}{\partial t^2}+\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\quad 0<x<1,\quad t>0,$$

subject to the conditions

$$u(0,t)=0,\quad u(1,t)=0,\quad t>0,$$

$$u(x,0)=f(x),\quad \frac{\partial u(x,0)}{\partial t}=0,\quad 0<x<1.$$

(b) Summarize the effects of the term $\partial u/\partial t$ on the left-hand side of the partial differential equation.

參考用

注意：背面有試題