＊請在試卷答案卷（卡）內作答

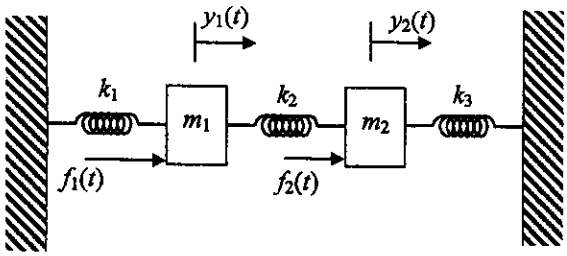
＊本科考試可使用計算器，廠牌、功能不拘

Ordinary Differential Equation (25 %)

1. Solve the nonhomogeneous Euler-Cauchy equation (25%)

$$x^3 y''' - x^2 y'' - 7xy' + 16y = x^3 \ln x$$

Laplace/Fourier Transformation (25 %)



2. A mass-spring system as shown in above figure can be modeled in terms of the following equations of motion,

$$\begin{aligned} m_1\ddot{y}_1 + (k_1+k_2)y_1 - k_2 y_2 &= f_1(t) \\ m_2\ddot{y}_2 - k_2 y_1 + (k_2+k_3)y_2 &= f_2(t) \end{aligned}, \quad y_1(0) = \dot{y}_1(0) = y_2(0) = \dot{y}_2(0) = 0,$$

where m_1 and m_2 represent the mass, k_1~k_3 are the spring constants, $f_1(t)$ and $f_2(t)$ are external forces, $y_1(t)$ and $y_2(t)$ represent the displacements of m_1 and m_2, respectively, and $\ddot{(\bullet)}$ represents $\dfrac{d^2(\bullet)}{dt^2}$.

(a) Suppose $f_1(t) = \begin{cases} 2; & 0 < t < 3 \\ 0; & t > 3 \end{cases}$, and $f_2(t) = \begin{cases} 0; & 0 < t < 2 \\ \delta(t-5); & 2 < t < 6 \\ 2te^{-3t}\cos(3\pi t); & t > 6 \end{cases}$. Let $\mathcal{F}_1(s)$ represents the Laplace transform of $f_1(t)$. Find $\mathcal{F}_1(s)$. (5%)

(b) Express the function of time, $f_1(t)$, in terms of functions of frequency, $\alpha(\omega)$ and $\beta(\omega)$, by using the Fourier Integral representation of $f_1(t)$. (7%)

(c) Let m_1=1, m_2=1, k_1=4, k_2=5/2, and k_3=4.

Find the solution $y_1(t)$ and $y_2(t)$ for $0 < t < 4$. (13%)

注意：背面有試題

參考用

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Linear Algebra (25 %)

3. Let C be the curve traversing the quarter-circle $x^2+y^2=1$ from $(1,0)$ to $(0,1)$ in the plane, then moving along the horizontal line from $(0,1)$ to $(2,1)$. Let $\vec{F}(x,y,z)=4x\vec{i}$. Compute $\int_C \vec{F}\cdot d\vec{R}$. (5%)

4. Consider the linear system equation $\mathbf{Ax}=\mathbf{b}$ where

$$\mathbf{A}=\begin{bmatrix}3&2&1\\2&1&1\\6&2&m\\5&3&2\end{bmatrix}\quad\text{and}\quad\mathbf{b}=\begin{bmatrix}3\\n\\6\\3+n\end{bmatrix}.$$

Let both m and n be real number, determine

(a) the rank of $\mathbf{A}$; (3%)

(b) the values of m and n, if the system equation has infinitely many solutions; (4%)

(c) the values of m and n, if the system equation has precisely one solution; and (4%)

(d) the values of m and n, if the system equation has no solution. (4%)

5. Please directly apply the theorem of Gauss, $\iiint_T \mathrm{div}\vec{F}dv=\iint_S \vec{F}\cdot\vec{n}dA$, to evaluate the integral $\iint_S (7x\vec{i}-z\vec{k})\cdot\vec{n}dA$ over the sphere $S: x^2+y^2+z^2=9$. (5%)

參考用

程式語言 (25 %)

6. 空間中三點 P1、P2、P3，其座標分別為 P1=(a1 b1 c1)、P2=(a2 b2 c2)、P3=(a3 b3 c3)。試寫一程式計算向量 v1=P2- P1 及 v2=P3-P1 的外積，將結果儲存於向量 v3。程式碼限定以 C、C++、Visual Basic 或 Fortran 撰寫，所有變數均以實數宣告，其數值無需考慮，並註明使用的程式語言。 (10%)

7. 有三個矩陣 A、B、C，維數分別為 5x4、4x3、5x3，若矩陣 A 與 B 為已知，並且 C=A x B，試寫一程式計算矩陣 C，程式中需使用迴圈計算，程式碼限定以 C、C++、Visual Basic 或 Fortran 撰寫，所有變數均以實數宣告，其數值無需考慮，並註明使用的程式語言。(15%)

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