1. Please explain and compare the following terms.
   \((5\%)\) a. Power series method and extended power series method (\textit{Frobenius} method);
   \((5\%)\) b. Orthogonality of vectors and orthogonality of functions;
   \((5\%)\) c. Laplace transform and Fourier transform;
   \((5\%)\) d. Unit step function and Dirac delta function;

2. Find orthogonal trajectories of the following function

\[ x^2 + (y - c)^2 = c^2 \]

\((10\%)\)

3. Solve the following initial value problem.
\[
(D^2 + 4D + 5I)y = e^{-t}\cos t \quad y(0) = 0, \quad y'(0) = 1
\]

\((15\%)\)


\[
\begin{bmatrix}
0 & 2 + 2i & 0 \\
2 - 2i & 0 & 2 + 2i \\
0 & 2 - 2i & 0
\end{bmatrix}
\]

\((15\%)\)

5. Evaluate \(\int_C \mathbf{F} \cdot \mathbf{r'} \, ds\), where \(C\) is the circle \(x^2 + y^2 = 4, \ z = -3\), oriented counterclockwise as seen by a person standing at the origin, and, with respect to right-handed Cartesian coordinates, \(\mathbf{F} = yi + xz^3j - zy^3k\).

\((15\%)\)

6. Please find the corresponding Fourier series of the following function.
\[ f(x) = x + \pi, \quad \text{if } -\pi < x < \pi \quad \text{and} \quad f(x + 2\pi) = f(x) \]

\((15\%)\)

7. Let \(\bar{u}\) and \(\beta\) be constant, find a plane wave solution of the following equation,

\[
\left( \frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) \left( \frac{\partial^3 \psi}{\partial x^2} + \frac{\partial^3 \psi}{\partial y^2} \right) + \beta \frac{\partial \psi}{\partial x} = 0
\]

\((10\%)\)