國立中央大學 107 學年度碩士班考試入學試題

所別： 大氣科學學系大氣物理 碩士班 不分組(一般生)
大氣科學學系大氣物理 碩士班 不分組(在職生)

科目： 應用數學

本科考試禁用計算器

＊請在答案卷(卡)內作答

1. Solve the following initial value problems.

   a. $y' + xy = xy^{-1}, \qquad y(0) = 3$

   b. $\begin{aligned} y_1' &= y_2 + 2e^t \\ y_2' &= y_1 - 2e^t \end{aligned} \qquad y_1(0) = 0, \quad y_2(0) = 1$

(30%)

2. Solve the following integral equation

$$y(t) + 2e^t \int_0^t y(\tau) e^{-\tau} d\tau = te^t$$

(15%)

3. Verify that $\mathbf{A}$ and $\hat{\mathbf{A}} = \mathbf{P}^{-1}\mathbf{AP}$ have the same spectrum.

$$\mathbf{A} = \begin{bmatrix} -4 & 6 & 6 \\ 0 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} 1 & 8 & -7 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

(15%)

4. Is the given function even or odd? Find its Fourier series.

$f(x) = 1 - x^2/4, \quad (-2 < x < 2)$ and $f(x+4) = f(x)$

(15%)

5. Find the Taylor series with center z_0 and its radius of convergence.

$\cos z, \quad z_0 = \pi$

(10%)

6. Let $f = xy - yz$, $\mathbf{v} = [4z \quad 2y \quad x - z]$, $w = [y^2 \quad y^2 - x^2 \quad 2z^2]$. Find

   a. ∇f at $P:(0, \ 3, \ 1)$ b. $\nabla^2(xzf)$ c. $\nabla \times \mathbf{v}$

   d. $\nabla \cdot \mathbf{w}$ e. $\mathbf{v} \cdot ((\nabla \times \mathbf{w}) \times \mathbf{v})$

(15%)