1. Solve the following initial value problems.
   a. \( y'' - 4y' + 3y = 10 \cos x, \quad y(0) = 1, \quad y'(0) = -1 \);
   b. \( (x^2 D^2 + xD - I)y = 16x^3, \quad y(1) = -1, \quad y'(1) = 1 \) (30%)

2. Find the inverse Laplace transform of the following functions
   a. \( \frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2} \)
   b. \( \frac{3s}{s^2 - 2s + 2} \) (10%)

3. Evaluate the following integral by the divergence theorem.
   \( \mathbf{F} = [ax \quad by \quad cz], \quad S \text{ the sphere } x^2 + y^2 + z^2 = 36 \) (15%)

   \[
   \begin{bmatrix}
   \frac{1}{4} & i\sqrt{2} \\
   i\sqrt{2} & \frac{1}{4}
   \end{bmatrix}
   \] (15%)

5. Find (a) the Fourier cosine series, (b) the Fourier sine series of the following function
   \( f(x) = \sin x, \quad 0 < x < \pi \). (15%)

6. Solve for \( u = u(x, y) \):
   \( u_{xx} + u_x = 0, \quad u(0, y) = f(y), \quad u_x(0, y) = g(y) \) (15%)