

1. Use residue calculus to evaluate the following integral

$$I = \int_0^{2\pi} \frac{d\theta}{A + B \cos \theta} \quad \text{for } A^2 > B^2 \text{ and } A > 0$$

(20%)

2. Use Laplace Transformation to solve the following initial value problem

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 2y = 0, \quad y(0) = 1, \quad \frac{dy(0)}{dt} = -1$$

(20%)

3. Determine the eigenvalues and eigenvectors of the following matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(20%)

4. Find a general solution of  $y'' + \omega^2 y = r(t)$ , where

$$r(t) = t^2/4, \quad (-\pi < t < \pi), \quad r(t+2\pi) = r(t)$$

(20%)

5. Evaluate  $\oint_C \frac{\partial w}{\partial n} ds$  counterclockwise around the boundary C of the region R,

where

$$w = (x+3y)^2 + 3x$$

$$R: \quad x^2 + y^2 \leq 16, \quad x \leq 0, \quad y \geq 0.$$

(20%)